Incentives and the Stock Market in General Equilibrium

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General Equilibrium: Problems, Prospects and Alternatives

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1 Recent Trends in General Equilibrium Theory

The objective of general equilibrium theory is to understand how the complex structure of contractual markets, characteristic of a modern economy, provides a mechanism for agents (consumers, firms and government) to coordinate their decisions, share their risks and create appropriate incentives, in an evolving intertemporal setting with uncertainty. The basic skeleton on which the theory is constructed is the classical theory, first envisioned as the felicitous "invisible hand" of Adam Smith (1776), enriched by the theoretical fabric contributed by Walras (1874) and Pareto (1909), and elegantly transformed into a mathematical framework some two hundred years later in the Arrow-Debreu theory (Debreu (1959)). This theory provides a highly idealized, abstract model of markets working at their best: the nature of the markets and the underlying contracts envisioned is austere and idealized in the extreme. For the Arrow-Debreu theory conceives a fictitious initial moment of time where all agents that are to live for the indefinite future assemble together to exchange contractual commitments, fully aware of all possible future scenarios, and fully confident that all the contractual commitments will be delivered in the future. The agents look up into the future—expressed as an immense date-event tree of possible scenarios, spelled out with meticulous detail and agreed upon by all agents—and in truly Olympian fashion, trade a complete set of Arrow-Debreu contracts, that rich family of promissory notes, each committing to deliver a good of carefully defined quality and characteristics at some future date-event: a truly grandiose thought experiment of uniquely ambitious proportions in the Social Sciences. The model maps all goods at all future date-events into the present, and assembles all future and present generations of agents onto a grand theatrical stage: it is clear that the model can not be taken literally as a description of reality—indeed some would argue that the whole problem with the model is that it is pure theater—this in essence is the argument of Gintis in this volume.

This is unfortunate for a fundamental insight of the Arrow-Debreu theory is that the co-ordination of productive activities and the exchange of goods and services between agents in an economy is effected through *contracts*, that is, promises made by one party to another to deliver a specified stream of goods and services from some initial date (the date of issue) until some specified future date (the date of maturity). Precisely because everything about the economy is so perfect—all agents are very knowledgeable and symmetrically informed, no agent ever acts opportunistically or defaults on his promises, being monitored by a perfect, costless legal system—society's problem of resource allocation can be solved by a very simple system of contracts all issued and signed at the initial date. The central insight of the model is that the resulting equilibrium outcome is "best" in a precise sense—being Pareto optimal—and every Pareto optimal allocation can always be achieved by such a complete system of Arrow-Debreu contracts.

The development of general equilibrium theory over the last thirty years can be viewed as an attempt to introduce various "imperfections" into the Arrow-Debreu description of an economy, for example, missing markets and asymmetry of information, retaining the idea that exchange and production is effected through contracts— but this time contracts that conform more closely with what we observe in the real world, including spot contracts and financial contracts such as bonds, equity and insurance. Building a theory which is consistent from top to bottom is a very difficult task. For example we have the intuition that when agents have limited knowledge about the actions and characteristics of other agents, and when the possibility arises that agents will behave opportunistically, then the Arrow-Debreu system essentially becomes unworkable. Agents would flee to the relative safety of spot markets where delivery is assured, making only limited forward trades through financial markets to smooth their income and diversify their risks. This explains why the subsequent generation of general equilibrium models has focused on sequential models, where agents trade at each date on spot markets and make limited forward commitments through financial markets.

One of the interesting properties of models with incomplete markets and asymmetry of information, is that they can often be viewed as constrained versions of an Arrow-Debreu model—and for analytical purposes studying them in this form usually leads to the most tractable mathematical formulation. As a result even though we do not use the Arrow-Debreu model directly as a descriptive model for the reasons just indicated, its canonical mathematical structure, and the tools and methods which were developed for analyzing it, reach far beyond the confines of the original model. This perhaps explains why there is still much active research on developing techniques for versions of the AD model in relatively abstract settings—for example with infinite dimensional commodity spaces (Mas-Colell-Zame (1991), Shannon (1999)) or in finance models without non-negativity constraints on consumption (Werner (1986), Page-Wooders (1996)), and Chichilnisky (1997) whose research is presented in this volume.

Sequential models. A significant part of the research on general equilibrium models in the last 30 years has been devoted to studying *sequential* models—the Arrow-Debreu assumption that all contracts are traded at an initial date being replaced by the more realistic assumption that there is trade at each date-event. Once the sequential nature of trade is admitted, two rather different ways present themselves for closing the model to obtain an equilibrium concept. To trade on markets today, agents must form expectations regarding prices and outcomes tomorrow. In a *temporary equilibrium* every agent is assumed to have an exogenously given, essentially arbitrary, expectation function regarding prices and outcomes tomorrow, and prices are sought which satisfy the minimal property of clearing the markets of today. When tomorrow arrives, the prices which clear the markets will typically prove that agents' expectations in the previous period regarding future prices were false (see Grandmont (1977) (1982)). The economy thus stumbles, as it were, through a sequence of false expectations. At the other end of the spectrum agents are assumed to find themselves in a market environment where they understand a great deal about what's going on: in a *correct* (*or rational*) *expectations equilibrium* agents correctly anticipate outcomes and prices tomorrow: not only do current prices clear the markets of today, but also the anticipated prices will clear the markets tomorrow in every possible contingency (see Radner (1972) (1982)).

The temporary equilibrium model was studied extensively in the 1970's and was viewed as a promising candidate for integrating Keynesian macroeconomic ideas with general equilibrium theory. The analytical and conceptual difficulties encountered in obtaining a more satisfactory theory of expectation formation, and the rather arbitrary nature of the results obtained when agents have exogenously given expectations, led the temporary equilibrium model to be essentially abandoned. This is unfortunate since the spirit of the equilibrium concept, namely that agents can hold different beliefs regarding future prices and payoffs, and that such differences of opinion provide an important motive for trade on financial markets, is surely correct as stylized fact.[1] In the language of Wall Street, a trader is at any moment either a bull or a bear, and the course of prices is importantly influenced by the proportion of bulls to bears. In an important recent contribution, Kurz (1994a,b)(1997) has introduced a concept of equilibrium with rational beliefs which permits agents to trade with such differences in beliefs: this equilibrium concept lies half way between a temporary and a rational expectations equilibrium—for rational beliefs are not required

[1] Keynes (1930, ch.15) was an articulate advocate of the idea that differences of opinion are an important explanatory factor for trade on financial markets.

to be correct but they are not arbitrary either, since they must be consistent with the realized frequencies of events as revealed in past data (see Kurz's contribution in this volume).

It is the concept of rational expectations equilibrium, however, which has become the predominant equilibrium concept for sequential models since the mid 1970's. The idea that rational agents trading on forward looking (speculative) markets will use available information to predict the future, and that they are not likely to be fooled into making systematic mistakes in their predictions, is compelling—and it is perhaps no accident that the areas of economics in which the concept of rational expectations has been felt to provide a good first approximation are precisely those in which there is most money at stake for those willing to make intelligent forecasts: the *efficient markets hypothesis* dominates all the modern theory of equilibrium on financial markets, and the idea that agents make use of their information regarding government monetary policy in forming their expectations of inflation has become a basic tenet of the rational expectations school of macroeconomics. We take no stand here as to the correctness or incorrectness of such models: what is certain, is that models with rational expectations have provided important new insights into the anticipatory behavior of security prices, and on the possible ineffectiveness of monetary policy in settings where agents can anticipate the monetary authority's future actions.

The hypothesis of rational expectations is an equilibrium concept which depends both on individual behavior (how agents form their expectations and make their decisions) and on market clearing (the determination of prices). It presupposes a setting where agents make informed predictions of future prices—correct predictions based on a correct model of the economy. The subsequent market clearing prices that arise from the decisions based on these expectations confirm their predictions. A rational expectations equilibrium is essentially the only equilibrium concept which is consistent in the sense that agents' expectations are self-fulfilling.

The basic building block for trade in a sequential general equilibrium model with uncertainty is the system of *spot markets* at the nodes of the underlying event-tree. As time and uncertainty unfold, the economy walks, as it were, along a path through the event-tree and, at any given node, an agent sells his endowment at this node and purchases a vector of goods at the current spot prices. Since agents buy and sell goods on current spot markets (at each node of the event-tree) each agent faces a sequence of budget constraints, and this sequence of budget constraints is the key distinguishing feature of the sequential model. Since agents' endowments and the outputs of firms (whose ownership is distributed among the agents) are subject to shocks, the agents will typically want to redistribute their income across the nodes of the event-tree, "borrowing and lending" to smooth their income over time, and buying or selling "rights to income streams" to diversify their risks across the nodes: this of course is the canonical role of the *financial markets*, for example the bond and equity markets, the commodity futures markets, derivative securities (puts and calls) and insurance markets. The richer the structure of these financial markets, the more readily agents can redistribute income across the event-tree: and when—at a cost—an agent can obtain any desired profile of income across the event-tree by the appropriate choice of a portfolio of the currently available securities at each node, the financial markets are said to be *complete*: roughly speaking what is required is that at each node there should be as many tradeable securities as there are immediately succeeding contingencies (the so-called *branching number* of the event-tree at that node). A sequential model with correct anticipations in which the financial markets are complete has the same equilibrium allocations as the Arrow-Debreu model (see Magill-Shafer(1990)).

The bulk of research on the sequential GE model in the 1980's and 1990's has been directed to exploring the consequences of a lack of complete markets for equilibrium allocations, a research agenda known as GEI, or *general equilibrium with incomplete markets.* In the discrete-time, discrete-state-space model which is commonly used, discontinuities in the demand for securities can occur when the security payoff matrix has a change of rank. This can happen when the payoffs depend on the spot prices (for example for futures contracts) or if the securities are long-lived so that the per-period payoff involves capital value terms (for example for equity)—and the discontinuities in demand can create non-existence of equilibrium. In essence every sequential GE model that involves either more than one good or more than two periods has a potential problem with non-existence of equilibrium. This problem, first uncovered by Hart (1975), discouraged work in the area for almost ten years: the problem was solved in the mid 1980's when it was proved that an equilibrium exists "generically", that is, for almost all endowments and security structures (Duffie-Shafer (1985), Hirsch-Magill-Mas-Colell (1990), Magill-Shafer (1991)). This result was a high point of mathematical economics since it involved beautiful mathematical arguments drawing on the powerful techniques of differential topology.[2] It established that the problem of non-existence would typically only arise for "exceptional" parameter values, so that research could justifiably focus on establishing qualitative properties of the equilibria.

Nominal assets and money. One of the striking attributes of the Arrow-Debreu model is that it describes the economic world entirely in real terms: there is no role and hence no room for money to affect the equilibrium outcome. As Hahn ((1971), (1973a,b)) has convincingly argued,

[2]For readers with a mathematical bent, it was shown that the underlying "fixed point" argument involves a more general theorem than the Brouwer fixed point theorem—essentially a "fixed point" theorem for maps from the Grassmannian of J-dimensional subspaces of $\mathbb{R}^S$ to $\mathbb{R}^{JS}$, for an economy with S states of nature and J securities (see Hirsch-Magill-Mas-Colell (1990)).

any attempt to introduce money into a general equilibrium model must be placed in the setting of a sequence economy. Once we enter a world of sequential markets, new possibilities arise for exploring the way money enters into the determination of an equilibrium. Modeling money in a satisfactory way in a general equilibrium model is a notoriously difficult task, and we are far from having even the elements of a monetary general equilibrium theory. Money is an asset which can be held like the other financial securities in a GEI model: however a distinct function must be assigned to money relative to the other financial assets if agents are to be induced to hold it, since typically its rate of return is dominated by those on other assets. The characteristic role of money, which distinguishes it from other assets, is that it serves as a medium of exchange. Magill-Quinzii (1992) have shown how, by introducing an appropriate transactions technology which formalizes Clower's idea that "money buys goods and goods buy money, but goods cannot buy goods", a real GEI equilibrium can be transformed into a monetary GEI equilibrium, in which a monetary authority decides at each date-event how much money to inject into the economy for use for transactions purposes. In essence a new equation is added at each node of the event-tree—over and above the equations equating demand and supply for each of the goods and each of the securities—which expresses equality between the demand for money for transactions purposes and the supply made available by the monetary authority. These quantity-theory equations determine the price level at each node of the event-tree.

Once such a transactions technology has been introduced, it becomes useful to distinguish between two types of financial securities. We say that a security is *real* if its payoff at any node is a linear function of the spot prices at this node: this is true for a commodity futures contract or an equity contract. A security is said to be *nominal* if its payoff at a node is an amount of money which is independent of the spot prices—most bonds are nominal assets. An important property of a real security is that it is inflation-proof—if the price level doubles at some node then its payoff at that node doubles—while for a nominal security, when the price level doubles, its purchasing power is cut in half. Monetary policy is said to be *neutral* if changing the monetary policy leaves the real equilibrium allocation unchanged. Building on results on the indeterminacy of an equilibrium allocation when nominal securities are introduced into an otherwise real GEI model (Balasko-Cass (1989) Geanakoplos-Mas-Colell (1989)), the following properties of a monetary equilibrium can be established (see Magill-Quinzii (1992), (1996)): if all securities are real, then regardless of the degree of incompleteness of the markets, monetary policy is neutral; if there are nominal securities and the financial markets are *complete*, then perfectly anticipated monetary policy is also neutral—in essence when agents anticipate a different monetary policy they rearrange their portfolios so as to offset the changed purchasing power of the nominal securities across the date-events; however if the financial markets are *incomplete* and there are nominal securities, then monetary policy, even when perfectly anticipated, is non-neutral. In essence, when monetary policy is changed agents do not have enough instruments at their disposal to compensate for the change in the purchasing power of the nominal securities across the date-events. This latter result can in turn be used to establish some simple stylized properties of an "optimal" monetary policy: for example it can be shown that, in an economy without fundamental uncertainty, monetary policy should not introduce "monetary shocks", i.e. be a new source of uncertainty for the agents in the economy.

Of course, it might be asked: why would agents ever want to hold nominal securities since it exposes them to fluctuations in the purchasing power of money? This is an old puzzle in monetary theory which has long seemed difficult to explain.[3] The GEI model of a monetary equilibrium can be used to throw light on this question. For once it is recognized that real shocks create fluctuations in the relative prices of goods across date-events, then we can understand why "indexing", while it eliminates the risks arising from fluctuations in the price level, necessarily introduces a new risk, namely that arising from the fluctuations in the *relative prices* of goods. Magill-Quinzii (1997) show that in an economy exposed to fluctuations in relative prices arising from normal technological and supply or demand shocks, then there is a critical level of the variability of purchasing power of money such that agents will prefer to hold a nominal rather than an indexed bond if the fluctuations in the purchasing power of money are less than this critical level—basically agents prefer a small exposure to price level fluctuations to the greater risk arising from changes in the relative prices of the goods in the index. Of course, when as in a number of Latin American countries, such as the Argentine or Brazil, fluctuations in price levels exceed the critical level, then agents prefer the indexed bond. These results show in a clear way some of the useful insights that can be obtained from the monetary version of the GEI exchange model.

The sequential model provides a natural setting for studying contracts—that is, promises to deliver a stream of goods or services from the date the contract is signed to its date of maturity. The financial contracts mentioned above represent a subset of these contracts, but the model could be greatly enriched by including a much broader array of contractual agreements between firms to deliver or accept delivery of goods, as well as labor contracts promising to render labor services. To the extent that such contracts embody nominal elements, fluctuations in monetary policy will have even more pervasive real effects in a production economy.

[3] As Shiller (1997) noted: "That the public should generally want to denominate contracts in currency units—despite all the evidence that it is not wise to do so . . . should be regarded as one of the great economic puzzles of all time."

Understanding incompleteness of markets. A property of the GEI model which has received much attention in the recent literature is the inefficiency property of equilibrium with incomplete markets. More precisely, as soon as we move out of the setting of a two-period one-good model (the basic model of a finance economy) into a world with more than two periods or more than two goods, if the markets are incomplete, not only is an equilibrium not typically Pareto optimal—which is to be expected if some markets are missing—but more surprisingly it is not even *constrained Pareto optimal* (Stiglitz (1982)), Geanakoplos-Polemarchakis (1986), Geanakoplos-Magill-Quinzii-Dreze (1990). This means that a planner, even if forced to respect the limited availability of instruments for transferring income across the date-events imposed by the incompleteness of the markets, can find better consumption, portfolio and production decisions than those which agents are induced to undertake through the sequential system of markets.

The intuition underlying this constrained inefficiency is best understood in the simplest case of a two-period exchange economy with many goods: if the planner changes agents' portfolios, then the agents will change their planned trades on the spot markets at the next date, causing the spot prices to change. If the financial markets are complete, then, since agents' rates of substitution are equalized at equilibrium, the increased utility of the winners from the price change will exactly compensate the decreased utility of loosers; however, when markets are incomplete, rates of substitution typically differ and a change of portfolios can be found such that, under the Hicks-Kaldor criterion, the winners can compensate the loosers, and there is social gain to the planner's intervention. In a multiperiod setting, or in a model with production, the argument is a bit more involved, but the intuition is the same: a change in decisions taken at date t influences the income distribution and the supply of goods at date $t+1$, and thus the prices of the goods and securities at date $t+1$. As a result, when rates of substitution differ because of missing markets, the planner can find a small reallocation such that the winners can compensate the loosers. In a market economy there are spillover effects across markets which by definition (of competitive equilibrium) agents do not take into account: with incomplete markets, this leads to co-ordination failure in the overall system of markets.

This result poses rather serious problems of interpretation for the GEI model: should it be interpreted to mean that government intervention is called for as soon as there are incomplete markets? This seems unlikely. For to be successful, such intervention would require the planner to have access to enormous amounts of information on agents' and firms' characteristics—information that may be difficult to come by, in view of the well-known difficulty of inducing truthful revelation of preferences by consumers and technology by firms, not to speak of the massive calculation problem involved. Would it not be better for the government to seek to "complete" the markets rather than intervening in an incomplete markets environment? To answer questions of this kind, the structure of the model and the possible sources of the incompleteness of markets need to be more explicitly specified. For example the private sector does not offer insurance against unemployment—fundamentally because of the problem of moral hazard and adverse selection involved in a labor contract. The government steps in, but offers only limited insurance since it also has to cope with the underlying incentive problem. To enrich the model and suggest appropriate normative analysis, we need to model more directly the factors which cause markets to be incomplete.

Contracts theorists often object to general equilibrium models on the grounds that they do not take into account the problems posed by asymmetry of information and by strategic (especially opportunistic) behavior in the contracting process that supports the exchange and production activities (see Gintis's critique of general equilibrium). Much of contract theory is essentially bilateral and strategic, and hence outside the normal purview of markets—or is at most set in highly simplified partial equilibrium models. Perhaps the line of division between the general equilibrium and contract theory camps has been drawn too severely and naively: the two approaches are complementary and a more productive research agenda would be to seek ways of reconciling the two approaches, retaining the competitive assumption of standard price-taking behavior with perfect information whenever it is a reasonable approximation, while invoking strategic behavior whenever asymmetries of information or differences in bargaining strengths of the parties make the standard competitive assumption inappropriate.

Adverse selection and moral hazard in GEI. Some progress has recently been made in modeling the functioning of financial markets in a general equilibrium setting when agents act opportunistically. As soon as the contracts which are traded promise delivery of goods or income in the future, there is a possibility that agents may renege on their promises. One of the important functions of the legal system is to ensure that contracts are respected—and the standard GEI model assumes that agents only make promises which they can and do fulfill. This however is clearly an idealization, for there are limits to what a legal system can achieve at reasonable cost in enforcing the repayment of debts. Moreover, even if it were possible to monitor agents' actions, it would not always be socially desirable to enforce complete repayment of debts, since a limited tolerance for default encourages innovation and risk taking. In this volume Geanakoplos presents recent research with his co-authors on the modeling of strategic default in a GEI model, under various assumptions on the enforcement mechanism— for example utility penalties imposed by the legal system and/or a collateral requirement. The analysis is based on the assumption that there is a large number of anonymous buyers and sellers on each contract, and that all buyers of "promises" (lenders) can foresee correctly the average repayment rate of the sellers (borrowers), given the enforcement mechanism.

Despite the presence of moral hazard created by the possibility of default, the contracts which are traded are *anonymous* and the risks are implicitly pooled by intermediaries. In a related paper, Bisin-Gottardi (1999) study the functioning of markets for anonymous insurance contracts in the presence of moral hazard or adverse selection. In these models, intermediaries pool risks for investors, so that the equilibrium price of each contract only depends on the average (aggregate) behavior (default rate, accident rate) which results from the strategic behavior of many individuals. Since the underlying equilibrium concept only exploits a minimal amount of information concerning the actions or characteristics of the agents, the resulting allocation can only hope to have limited efficiency properties.

Our analysis, which we outline in the remainder of the paper, has focused on the other extreme of *named contracts* where the issuer of the contract for which there is a problem of moral hazard is a single legal entity, namely a corporation. In this setting the buyers of the contract have access to much more detailed information regarding the actions of the corporation which influence the payoff of its security, and as a result we show that the market acts as a disciplining device, attenuating the problem of moral hazard: the equilibrium price, instead of reflecting the average behavior of a pool of agents, becomes a complex equilibrium pricing function which reveals to the corporation the consequences for the price of its security of a whole family of out-of-equilibrium actions which it could have taken.

Introducing the moral hazard problem of corporate management into a general equilibrium framework permits two opposing views on the merits of the stock market to be studied in a common framework. In one view, the merit of the stock market is that it permits the substantial production risks of society to be diversified among many investors: this view underlies the capital asset pricing model (CAPM) which forms the basis for much of the modern theory of finance. On the other hand, the traditional view of classical economists (Adam Smith (1776), Mill (1848), Marshall (1890), revived in modern times by Berle and Means (1932), Jensen and Meckling (1976) and the ensuing agency-cost literature, has emphasized the negative effect on incentives of the separation of ownership and control implied by the corporate form of organization. The objective of the analysis which follows is to show how these two approaches can be captured in a general equilibrium model, and to discover the circumstances under which capital markets can provide an optimal trade-off between the beneficial effect of risk diversification and the distortive effect on incentives.

2 Incentives versus Risk-Sharing in Capital Markets

2.1 The basic model

To capture the dual role of capital markets in affecting risk sharing and incentives, we consider a simple general equilibrium model of a finance economy with production. In the spirit of Knight (1921) we model the firm as an entity arising from the organizational ability, foresight and initiative of an *entrepreneur*. The activity of a firm consists in combining entrepreneurial effort and physical input (the value of capital and non-managerial labor) at an initial date: this gives rise to a random profit stream at the next date. In addition to entrepreneurs there is another class of agents which we call *investors*: they have initial wealth at date 0 but no productive opportunities.

More formally consider a two-period one-good economy with production in which investment of both capital and effort at date 0 is required to generate output at date 1, the output at date 1 being uncertain. There are two types of agents in the economy, *entrepreneurs* and *investors*: $\mathcal{I}_1 \neq \emptyset$ is the set of entrepreneurs, $\mathcal{I}_2 \neq \emptyset$ the set of investors and $\mathcal{I} = \mathcal{I}_1 \cup \mathcal{I}_2$ is the set of all agents, which is assumed to be finite.[4] Every agent $i \in \mathcal{I}$ has an initial wealth $\omega_0^i > 0$ at date 0. If agent i is an entrepreneur, then by investing capital $\kappa^i \in \mathbb{R}_+$ (an amount of the good (income)) and effort $e^i \in \mathbb{R}_+$ at date 0 he can obtain the uncertain stream of income

$$\boldsymbol{F}^i(\kappa^i, e^i) = (F_1^i(\kappa^i, e^i), \ldots, F_S^i(\kappa^i, e^i))$$

at date 1, where $\mathcal{S} = \{1, \ldots, S\}$ is the set of states of nature describing the uncertainty and $\boldsymbol{F}^i : \mathbb{R}_+^2 \longrightarrow \mathbb{R}_+^S$ is a concave, differentiable, non-decreasing function, with $\boldsymbol{F}^i(\kappa^i, 0) = \boldsymbol{F}^i(0, e^i) = \mathbf{0}$. Investors are agents who do not undertake productive ventures, so if $i \in \mathcal{I}_2$, then $\boldsymbol{F}^i(\kappa^i, e^i) \equiv \mathbf{0}$.

Each agent has a utility function $U^i : \mathbb{R}_+^{S+1} \times \mathbb{R}_+ \longrightarrow \mathbb{R}$, where $U^i(\boldsymbol{x}^i, e^i)$ is the utility associated with the consumption stream $\boldsymbol{x}^i = (x_0^i, x_1^i, \ldots, x_S^i)$ and the effort level e^i. The utility function is assumed to be separable

$$U^i(\boldsymbol{x}^i, e^i) = u_0^i(x_0^i) + u_1^i(x_1^i, \ldots, x_S^i) - c^i(e^i)$$

where the functions u_0^i, u_1^i are differentiable, strictly concave, increasing, and c^i is differentiable, convex, increasing, with $c^i(0) = 0$.

To ensure that the technology of each entrepreneur i is sufficiently productive to make it worthwhile to operate, we assume that, as soon as entrepreneur i invests some positive effort in his firm,

[4]ßSets are denoted by calligraphic letters: the same letter in roman denotes the cardinality of the set, e.g. $\mathcal{I} = \{1, \ldots, I\}$. Vectors, matrices and vector-valued functions are written in boldface.

the marginal productivity of capital at zero is infinite. More precisely we assume that for all $i \in \mathcal{I}_1$ there is a smooth path $e^i : [0,1] \longrightarrow \mathbb{R}_+$ with $e^i(0) = 0$ and $e^{i\prime}(t) > 0$ such that

$$\lim_{t \to 0} c^{i\prime}(e^i(t))e^{i\prime}(t) < \infty \quad \text{and} \quad \lim_{t \to 0} \frac{\partial F_s^i}{\partial z^i}\Big(t, e^i(t)\Big) = \infty, \quad \text{for some } s \in \mathcal{S}.$$

It is easy to see that this assumption implies that for all $\boldsymbol{x}^i = (x_0^i, \boldsymbol{x}_\mathbf{1}^i) \in \mathbb{R}_+^{S+1}$ with $x_0^i > 0$, there exist $(\kappa^i, e^i) \gg 0$ such that

$$u_0^i(x_0^i - \kappa^i) + u_1^i(\boldsymbol{x}_\mathbf{1}^i + \boldsymbol{F}^i(\kappa^i, e^i)) - c^i(e^i) > u_0^i(x_0^i) + u_1^i(\boldsymbol{x}_\mathbf{1}^i)$$

so that even if there were no market to finance the capital investment, entrepreneur i would choose to produce. To bound the economy we assume that, for any positive level of capital input, the marginal cost of effort eventually exceeds its marginal product

$$\frac{\partial \boldsymbol{F}^i(\kappa^i, e^i)}{\partial e^i} \to 0, \quad \text{and} \quad c^{i\prime}(e^i) \to \infty \;\; \text{when } e^i \to \infty, \quad i \in \mathcal{I}_1$$

This implies that, for a given level of capital, the effort chosen by entrepreneur i will always remain bounded.

2.2 An Ideal World: Sole Proprietorship with Arrow Securities

In this model there is a moral hazard problem when two imperfections are present simultaneously. The first is that *effort is not observable*, so that contracts cannot be written contingent on the effort invested by entrepreneurs in their firms. This would not create a problem without the constraint that *states of nature are not verifiable*, so that the enforcement of contracts contingent on states is not feasible.

To see this, let us imagine an ideal world in which financial contracts could be written contingent on the states of nature, so that the securities have payoffs which are independent of the agents' actions, and suppose in addition that such contracts are complete —in short that there is a complete set of Arrow securities. Then there would be a simple way of obtaining a Pareto optimal allocation, despite the non-observability of effort. It would suffice to let each entrepreneur be the *sole proprietor* of his firm so that he has both the full marginal benefit and cost of his effort and there is no distortion of incentives: to adjust the risk profile of his income stream, he trades Arrow securities whose payoffs are independent of his actions. To make these statements precise, let us introduce the concept of a sole-proprietorship equilibrium with Arrow securities. Letting the price of income at date 0 be normalized to 1 and letting π_s denote the price (at date 0) of the Arrow security for state s (which delivers one unit of income in state $s \in \mathcal{S}$), the budget set of agent i with Arrow securities and sole proprietorship is given by

$$B(\boldsymbol{\pi}, \omega_0^i, \boldsymbol{F}^i) = \left\{ (\boldsymbol{x}^i, e^i) \in \mathbb{R}_+^{S+2} \;\middle|\; \begin{array}{l} x_0^i = \omega_0^i - \boldsymbol{\pi}\boldsymbol{\zeta}^i - \kappa^i \\ \boldsymbol{x}_{\mathbf{1}}^i = \boldsymbol{\zeta}^i + \boldsymbol{F}^i(\kappa^i, e^i) \\ (\kappa^i, \boldsymbol{\zeta}^i) \in \mathbb{R}_+ \times \mathbb{R}^S \end{array} \right\}$$

where $\boldsymbol{\pi} = (\pi_1, \dots, \pi_S)$ and $\boldsymbol{\zeta}^i = (\zeta_1^i, \dots, \zeta_S^i)$ is agent i' portfolio of the Arrow securities. As usual, the $S + 1$ budget constraints with Arrow securities can be reduced to a single budget constraint, i.e. the set $B(\boldsymbol{\pi}, \omega_0^i, \boldsymbol{F}^i)$ can be written as

$$B(\boldsymbol{\pi}, \omega_0^i, \boldsymbol{F}^i) = \left\{ (\boldsymbol{x}^i, e^i) \in \mathbb{R}_+^{S+2} \mid x_0^i + \boldsymbol{\pi}\boldsymbol{x}_{\mathbf{1}}^i = \omega_0^i + \boldsymbol{\pi}\boldsymbol{F}^i(\kappa^i, e^i) - \kappa^i \right\} \tag{1}$$

This is the budget set of an agent with contingent markets for income and sole proprietorship, expressing equality of the present value of the agents' lifetime expenditure and income.

Definition 1. $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}}; \bar{\boldsymbol{\pi}})$ is *a sole-proprietorship equilibrium with Arrow securities* (SPA) if

(i) $(\bar{\boldsymbol{x}}^i, \bar{e}^i, \bar{\kappa}^i) \in \arg\max \left\{ U^i(\boldsymbol{x}^i, e^i) \;\middle|\; (\boldsymbol{x}^i, e^i) \in B(\bar{\boldsymbol{\pi}}, \omega_0^i, \boldsymbol{F}^i) \right\}$ and $\bar{\boldsymbol{y}}^i = \boldsymbol{F}^i(\bar{\kappa}^i, \bar{e}^i),\ i \in \mathcal{I}$

(ii) $\sum_{i \in \mathcal{I}} x_0^i = \sum_{i \in \mathcal{I}} \omega_0^i - \sum_{i \in \mathcal{I}} \kappa^i, \qquad \sum_{i \in \mathcal{I}} \boldsymbol{x}_{\mathbf{1}}^i = \sum_{i \in \mathcal{I}} \boldsymbol{F}^i(\kappa^i, e^i)$

An SPA is not precisely an Arrow-Debreu equilibrium, since there are $S + 1 + I_1$ "goods" in the economy — the $S + 1$ incomes at dates 0 and 1, and the I_1 effort levels of the entrepreneurs — but there are only $S + 1$ markets. Despite the absence of the I_1 markets for the effort levels of entrepreneurs, the first and second welfare theorems — as well as the existence of equilibrium — are satisfied by SPA equilibria. This is due to the following two properties of "Robinson Crusoe" economies:

(i) An agent who is both a producer and a consumer in a convex economy can be split into two "personalities": an entrepreneur who maximizes profit and a consumer who takes the profit as given and maximizes utility over his budget set (see Magill-Quinzii (1996) for an account of this property in a general framework).

(ii) Agent i as the entrepreneur running firm i buys the input "effort for firm i" from only one agent, himself as a consumer. The market for effort e^i can thus be "internalized" in the joint consumer-producer maximum problem of agent i in a SPA. Any other ownership structure of the firm would fail to lead to Pareto optimality in the absence of a market for effort.

Proposition A (Properties of SPA equilibrium).

(i) *For any* $\boldsymbol{\omega}_0 \in \mathbb{R}^I_+$, $\omega_0 \neq 0$, *there exists an SPA equilibrium.*

(ii) *If* $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}}; \bar{\boldsymbol{\pi}})$ *is an SPA equilibrium, then the allocation* $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}})$ *is Pareto optimal.*

(iii) *For any Pareto optimal allocation* $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}})$ *there exist incomes* $\boldsymbol{\omega}_0 \in \mathbb{R}^I$ *and state prices* $\bar{\boldsymbol{\pi}} \in \mathbb{R}^S_{++}$ *such that* $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}}; \bar{\boldsymbol{\pi}})$ *is an SPA equilibrium.*

The existence proof is standard: to prove the equivalence between PO allocations and SPA equilibria in the differentiable case it suffices to note that the FOC for Pareto optimality are the same as the FOC for the maximum problems of the agents in an SPA equilibrium:

$$\frac{\partial u_1^i(\bar{\boldsymbol{x}}_{\boldsymbol{1}}^i)}{\partial x_s^i} / u_0^{i\prime}(\bar{x}_0^i) = \bar{\pi}_s, \quad c^{i\prime}(e^i) = \sum_{s \in \mathcal{S}} \bar{\pi}_s \frac{\partial F_s^i}{\partial e^i}(\bar{\kappa}^i, \bar{e}^i), \quad 1 = \sum_{s \in \mathcal{S}} \bar{\pi}_s \frac{\partial F_s^i}{\partial \kappa^i}(\bar{\kappa}^i, \bar{e}^i), \quad i \in \mathcal{I}$$

In both cases the problems are convex so that the FOC are necessary and sufficient.

Proposition A asserts that in an ideal world where states of nature are verifiable, society's problem of resource allocation—even when faced with the problem of non-observability of effort of the entrepreneurs who run the economy's firms—can be solved by letting entrepreneurs be sole proprietors of their firms and permitting agents to transfer income by trading Arrow securities. This structure of ownership and markets solves the twin problem of incentives and risk sharing by keeping them separate: sole proprietorship creates incentives and Arrow securities induce optimal risk sharing.

2.3 The Real World: Moral Hazard with Output-Contingent Contracts

In practice few insurance contracts based on "primitive causes" exist for sharing the risks of business. For most of the important risks which influence a firm's profits—shocks of varying magnitude to demand, to technology, to the competitive environment, and to input availability, as described in approximate terms in corporate quarterly and annual reports to shareholders—*no insurance contracts are available.* To an experienced businessman this is obvious—for it is the essence of business that such risks cannot be insured.[5] Given the difficulty of describing precisely ex-ante and verifying accurately ex-post the precise nature of the primitive shocks, most financial contracts that are used for financing firms and sharing productive risks are either non-contingent (bonds) or based directly on the realized *outputs* of firms (equity and derivative securities, like options on equity). To reflect these restrictions imposed by the real world on the nature of the contracts that can be

[5] This is the central theme of Knight's (1921) classic treatise *Risk, Uncertainty and Profit.*

used for facilitating exchange and production, let us assume that only financial contracts based on the realized outputs of firms are feasible. To make the model coherent, we must however assume that investors and entrepreneurs *understand* the basic nature of the uncertainty (the shocks) to which firms are subjected.

Trading contracts contingent on realized output when there is no separate market for "effort" is liable however to lead to inefficiencies. For typically entrepreneurs need funds to finance their projects. If they only have access to borrowing (debt), they will be exposed to rather risky leveraged positions, and if there are penalties for default, then they will have to restrict the amount they borrow for fear of bad contingencies. One traditional remedy lies in introducing the possibility of financing by issuing equity: this additional source of funds permits entrepreneurs to share the risks involved in their productive activity with investors, and opens up the possibility for all agents to diversify their risks. But selling ownership shares of his firm has a negative incentive effect on the entrepreneur, since any increment to effort is no longer rewarded by the full value of its marginal product.

Debt and equity contracts however constituted only the first stage in the development of financial markets to meet the financing needs of firms: at a later stage financial markets became more sophisticated, permitting the introduction and systematic use of derivative securities. Such securities serve two roles: they increase the risk-sharing possibilities of agents (the span of the markets) and provide instruments for creating incentives for managers of firms. Holding an option which is worthless unless the profit of the firm exceeds the striking price of the option provides a strong incentive for a manager to exert the extra effort needed to assure that the profit stream is likely to surpass this level. The extraordinary increase in the use of options as incentive devices in the last ten years in the US comes from the recognition of their great power as instruments for motivating CEO's. The use of options can potentially restore some, or, as we shall see in Section 5, even all of the incentives of entrepreneurs lost in reducing their equity shares to finance their firms.

The financial markets. The financial contracts which the agents in the economy can trade are thus taken to be: first, the default-free bond with (date 0) price q_0 and payoff stream $\mathbf{1} = (1, \dots, 1)$ at date 1; second for each firm $i \in \mathcal{I}_1$, its equity contract with price q^i_y and date 1 payoff stream $\boldsymbol{y}^i = (y^i_1, \dots, y^i_S)$; finally for each firm a family of derivative securities on its equity, consisting of call options with different striking prices. Let $\mathcal{J}^i \subset \mathbb{N}$ denote the index set for the call options on the equity of firm i, and let $\boldsymbol{\tau}^i = (\tau^i_j)_{j \in \mathcal{J}^i}$ denote the vector of associated striking prices, with $\boldsymbol{\tau}^i \in \mathbb{R}^{J^i}_+$. The call option (i, j) — the j^{th} option of firm i — has the price q^i_j at date 0 and the payoff stream $\boldsymbol{R_j^i} = (R^i_{j,1} \ldots, R^i_{j,S})$ across the states at date 1 given by

$$R^i_{j,s} = \max\{y^i_s - \tau^i_j, 0\}, \quad s \in \mathcal{S}$$

where $y^i_s = F^i_s(\kappa^i, e^i)$ denotes the output of firm i in state s. When it is important to stress that the choice of (κ^i, e^i) influences the payoff of the equity and thus of the option, we use the notation

$$R^i_{j,s}(\kappa^i, e^i) = \max\{F^i_s(\kappa^i, e^i) - \tau^i_j, 0\}, \quad s \in \mathcal{S}$$

Let $\boldsymbol{R}^i$ (or $\boldsymbol{R}^i(\kappa^i, e^i)$) denote the $S \times J^i$ matrix of payoffs of the J^i options of firm i, $\boldsymbol{R}^i_s$ the row vector of payoffs of the J^i options on firm i in state s and $\boldsymbol{R}^i_j$ the column vector of payoffs of option j across the states. Let $\mathcal{J} = \cup_{i\in\mathcal{I}_1}\mathcal{J}^i$ denote the set of all options and let $\boldsymbol{\tau} = (\boldsymbol{\tau}^i)_{i\in\mathcal{I}_1}$ denote the associated striking prices. The economy with characteristics $\boldsymbol{U} = (U^i)_{i\in\mathcal{I}}$, $\boldsymbol{\omega}_0 = (\omega^i_0)_{i\in\mathcal{I}}$, $\boldsymbol{F} = (\boldsymbol{F}^i)_{i\in\mathcal{I}_1}$ for the agents, and with a market structure composed of the riskless bond, the equity contracts of the I_1 firms, and the set of options $\mathcal{J}$ with striking prices $\boldsymbol{\tau}$, will be denoted $\mathcal{E}(\boldsymbol{U}, \boldsymbol{\omega}_0, \boldsymbol{F}, \boldsymbol{\tau})$. In such an economy, we let $\boldsymbol{q}_y = (q^i_y)_{i\in\mathcal{I}_1}$ denote the vector of equity prices, $\boldsymbol{q}^i_c$ the vector of prices of the J^i call options of firm i, $\boldsymbol{q}^i = (q^i_y, \boldsymbol{q}^i_c)$ the vector of prices which are influenced by the actions of entrepreneur i, and $\boldsymbol{q} = (q_0, (\boldsymbol{q}^i)_{i\in\mathcal{I}})$ the vector of all security prices.

The agents' accounts. To simplify the analysis we assume that the penalty for default for individual agents and for bankruptcy by firms is infinite, so that the personal debt of an entrepreneur and the debt incurred by his firm are both default-free debt. With no default and no bankruptcy there is no loss of generality in assuming that the entrepreneur is personally responsible for the debt of his firm: thus the accounts of the firm and the entrepreneur can be lumped together, leading to a considerable simplification of the model.

At date 0 entrepreneur i decides on the amount of capital κ^i to invest in his firm, on the amount to borrow ξ^i_0 (if $\xi^i_0 > 0$, lend if $\xi^i_0 < 0$), and on the share $(1-\theta^i_i)$ of his firm to sell. He also purchases shares θ^i_k of other firms $k \neq i$, as well as amounts $\xi^i_{k,j}$ of the options of these firms ($j \in \mathcal{J}^k, k \neq i$) to diversify his risks. The purchase of a portfolio of options $(\xi^i_{i,j})_{j\in\mathcal{J}^i}$ on his own equity contract serves as an incentive device to "bond" his personal interest to those of the outside shareholders of his firm. Let $\boldsymbol{\theta}^i = (\theta^i_k)_{k\in\mathcal{I}_1}$, denote the equity portfolio, $\boldsymbol{\xi}^i_k = (\xi^i_{k,j})_{j\in\mathcal{J}^k}$ the portfolio of options of firm k and $\boldsymbol{\xi}^i = (\xi^i_0, (\xi^i_k)_{k\in\mathcal{I}_1})$ the portfolio of all securities in zero net supply (bond and options) held by agent i. If entrepreneur i anticipates the production $(\boldsymbol{y}^k)$ of other entrepreneurs, then a choice of the financial variables $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$, in conjunction with a choice of effort e^i, leads to a vector of consumption $\boldsymbol{x}^i = (x_0^i, x_1^i, \dots, x_S^i)$ given by

$$x_0^i = \omega_0^i - q_0\xi_0^i - \sum_{k\neq i} q_y^k \theta_k^i \;-\; \sum_{k\neq i} \boldsymbol{q}_c^k\, \boldsymbol{\xi}_k^i + q_y^i(1-\theta_i^i) - \boldsymbol{q}_c^i \boldsymbol{\xi}_i^i - \kappa^i \tag{2}$$

$$x_s^i = \xi_0^i + \sum_{k\neq i} \theta_k^i y_s^k + \sum_{k\neq i} \boldsymbol{R}_s^k \boldsymbol{\xi}_k^i + F_s^i(\kappa^i, e^i)\theta_i^i + \boldsymbol{R}_s^i(\kappa^i, e^i)\boldsymbol{\xi}_i^i, \quad s \in \mathcal{S} \tag{3}$$

If agent i is an investor, then the budget equations are the same with $\kappa^i = 0, e^i = 0, \boldsymbol{F}^i = \boldsymbol{0}, q_y^i = q_j^i = 0$, so that the terms related to his own "firm" are just dummy variables.[6]

It is clear from equations (3) that the date 1 reward of an entrepreneur for his effort depends on his choice of financial variables $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$. This captures the idea that the capital structure of a firm (in particular the inside equity and options held by the manager, and the firm's debt) affects the performance of its management. Since financing arrangements must be in place before a firm can become operational, we assume that the choice of effort e^i by an entrepreneur is made after the financial decisions $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$ have been determined. To make this sequential structure of decision making explicit, suppose that date 0 is divided into two subperiods, $0_1, 0_2$. In subperiod 0_1 entrepreneurs use the financial markets to obtain the capital required to set up their firms and diversify their risks: in the second subperiod 0_2, after the investment and financing decisions have been made, firms become "operational" and entrepreneurs decide on the appropriate effort to invest in running their firms. At date 1 "nature" chooses a state of the world $s \in \mathcal{S}$: production takes place and profit is realized.

How an entrepreneur decides his optimal effort. After entrepreneur i has chosen his financial variables $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$ in subperiod 0_1 (in a way that we will study shortly), in subperiod 0_2 he chooses the effort level e^i which maximizes $u_1^i(\boldsymbol{x}_{\boldsymbol{1}}^i) - c^i(e^i)$, where $\boldsymbol{x}_{\boldsymbol{1}}^i = (x_1^i, \dots, x_S^i)$ is the date 1 consumption stream given by (3). Entrepreneur i's financial variables are of two kinds: *inside* variables (those internal to the firm) which directly affect the payoff (reward) of the entrepreneur from his effort, and the *outside* variables (external to the firm) which determine the income that agent i gets independently of his effort. $(\kappa^i, \theta_i^i, \boldsymbol{\xi}_i^i)$ are the inside financial variables which determine his *inside income* (the last two terms in (3)), while $(\xi_0^i, (\theta_k^i, \boldsymbol{\xi}_k^i)_{k\neq i})$ are the outside variables which determine his *outside income* $\boldsymbol{m}^i = (m_1^i, \dots, m_S^i)$ defined by

$$\boldsymbol{m}^i = \boldsymbol{1}\,\xi_0^i + \sum_{k\neq i} (\boldsymbol{y}^k\,\theta_k^i + \boldsymbol{R}^k \boldsymbol{\xi}_k^i) \tag{4}$$

[6]ßWhen we need unified notation for both types of agents, entrepreneurs and investors, we adopt the convention that for $k \in \mathcal{I}_2$, $\theta_k^i = 0$ if $i \neq k$, $\theta_k^k = 1$ and $\mathcal{J}^k = \emptyset$.

namely the first three terms in (3). Define the *effort correspondence* of entrepreneur i

$$\tilde{e}^i(\boldsymbol{m}^i, \kappa^i, \theta^i_i, \boldsymbol{\xi}^i_i) = \arg\max_{e^i \geq 0} \left\{ u^i_1(\boldsymbol{x}^i_{\mathbf{1}}) - c^i(e^i) \,\middle|\, \boldsymbol{x}^i_{\mathbf{1}} = \boldsymbol{m}^i + \boldsymbol{F}^i(\kappa^i, e^i)\theta^i_i + \boldsymbol{R}^i(\kappa^i, e^i)\boldsymbol{\xi}^i_i \right\} \tag{E}$$

Since we have assumed that the marginal productivity of effort tends to zero when effort tends to infinity while its marginal cost tends to infinity, the *effort choice problem* (E) has a maximum for some finite value of e^i and the correspondence $\tilde{e}^i$ is well-defined on the domain $\mathcal{D}^i \subset \mathbb{R}^S_+ \times \mathbb{R}_+ \times \mathbb{R}^J \times \mathbb{R}^{J^i}$ consisting of the variables $(\boldsymbol{m}^i, \kappa^i, \theta^i_i, (\xi^i_{i,j})_{j \in \mathcal{J}^i})$ such that $\boldsymbol{m}^i + \theta^i_i \boldsymbol{F}^i(\kappa^i, e^i) + \sum_{j \in \mathcal{J}^i} \xi^i_{i,j} \boldsymbol{R}^i_j(\kappa^i, e^i) \in \mathbb{R}^S_{++}$ for some $e^i > 0$. In the special case where there are no options ($\mathcal{J}^i = \emptyset$), the assumptions of strict concavity of u^i_1, convexity of c^i and concavity of $\boldsymbol{F}^i$ ensure that the solution to (E) is unique, so that $\tilde{e}^i$ is a function on $\mathcal{D}^i$. When $\mathcal{J}^i \neq \emptyset$, the payoffs of the options introduce a non-convexity into the constraint set in E, and the solution of the maximum problem may not be unique: in this case $\tilde{e}^i$ is a correspondence defined on $\mathcal{D}^i$.

Equilibrium with rational competitive price perceptions. Consider an investor who is thinking of buying either the equity or options of firm i. To anticipate what the firm's profit will be, the investor needs to anticipate the entrepreneur's inputs (κ^i, e^i). In this model we assume that the capital input κ^i is observable, while the effort e^i is not. However, as we have seen, e^i can be deduced if the entrepreneur's characteristics $(u^i_1, \boldsymbol{F}^i, c^i)$ and his financial variables, or more precisely his outside income $\boldsymbol{m}^i$ and the inside financial variables $(\kappa^i, \theta^i_i, \boldsymbol{\xi}^i_i)$, are known: in the analysis that follows we assume that investors do indeed have access to this information and hence can deduce the effort e^i that the entrepreneur will invest in his firm.

In practice there is an important distinction between accessibility of information regarding the inside financial variables $(\kappa^i, \theta^i_i, \boldsymbol{\xi}^i_i)$ and information regarding the outside wealth $\boldsymbol{m}^i$ and characteristics $(u^i_1, \boldsymbol{F}^i, c^i)$ of a firm's manager. Disclosure rules of the Securities and Exchange Commission require that proxy statements of publicly traded firms contain information regarding capital projects of the firm, as well as the equity and options holdings of the top management. Thus the assumption that inside variables are known by investors conforms with the regulations of capital markets in the US.

More detailed information regarding the characteristics of the firm and its manager are less directly accessible, and it is essentially the job of security analysts to gain access to this type of information. While this information may not be available with the precision required by the model, analysts will however in the course of scrutinizing the earnings prospects of the firms they follow, acquire a good knowledge of the characteristics of the firms and their top management. Analysts who have followed the careers of top executives are likely to have a good estimate of the magnitude of their personal wealth and hence can impute at least the orders of magnitude of their outside incomes. Past performance gives information on their ability — which in the model is included in the function $\boldsymbol{F}^i$ — and their motivation and ability to take risks — in the model, the functions u_1^i and c^i. The information collected by analysts spreads to investors through advisory services and the recommendations given by large brokerage companies. The assumption that the characteristics and financial trades of the entrepreneurs are known by all agents is thus the theoretical limit of a situation in which both the rules of disclosure and the activity of professionals in financial services result in a large amount of information being available to investors in the market.

If entrepreneurs' financial trades are known to investors, if investors make optimal use of this information to anticipate the outputs of firms, and in this way come to decide on the prices they are prepared to pay for the equity and options of the firms, then it seems reasonable to suppose that entrepreneurs will come to understand this. Hence our second assumption regarding anticipations: entrepreneurs are aware that investors will use their financial decisions as "signals" of the effort that they will exert in their firms. The next step is to incorporate these two assumptions — namely that (1) *investors use the available information (the financial variables) to correctly anticipate the firms' outputs*, (2) *entrepreneurs understand this* — into a concept of equilibrium.

The description of an equilibrium thus consists of two parts. The first is the standard part which enumerates the *actions* of the agents and the *prices* of the securities; the second part describes the entrepreneurs' *perceptions* of the way their financial decisions affect the price that the "market" will pay for the securities — equity and options — based on the profit of their firm. Let $\tilde{\boldsymbol{Q}}^i = (\tilde{Q}_y^i, \tilde{\boldsymbol{Q}}_c^i) = (\tilde{Q}_y^i, (\tilde{Q}_j^i)_{j \in \mathcal{J}^i})$ denote the price perception of entrepreneur i where

$$\tilde{Q}_\beta^i : \mathbb{R}_+ \times \mathbb{R}^I \times \mathbb{R}^J \longrightarrow \mathbb{R}_+, \ \beta = y \text{ or } j, \ j \in \mathcal{J}^i$$

is the price that entrepreneur i expects for security β (his equity, or an option on his equity) if he chooses the financial variables $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$. Let $\tilde{\boldsymbol{Q}} = (\tilde{\boldsymbol{Q}}^1, \ldots, \tilde{\boldsymbol{Q}}_1^I)$ denote the price perceptions of all entrepreneurs.

It is useful to define the following date 1 payoff matrices associated with the different securities in the economy. Let $\boldsymbol{V}^0 = (1, \ldots, 1)^T$ be the payoff of the riskless bond and, for a vector $\boldsymbol{y} = (\boldsymbol{y}^i)_{i \in \mathcal{I}_1}$, let $\boldsymbol{V}^i(\boldsymbol{y}) = [\boldsymbol{y}^i, \boldsymbol{R}^i(\boldsymbol{y}^i)]$ denote the $S \times (1 + J^i)$ matrix of payoffs of the securities of firm i. $\boldsymbol{V}(\boldsymbol{y}) = [\boldsymbol{V}^0, \boldsymbol{V}^1(\boldsymbol{y}), \ldots, \boldsymbol{V}^{I_1}(\boldsymbol{y})]$ denotes the $S \times [1 + (1 + J^1) + \ldots + (1 + J^{I_1})]$ payoff matrix of all the securities and $\boldsymbol{V}_{-i}(\boldsymbol{y}) = [\boldsymbol{V}^0, \ldots, \boldsymbol{V}^{i-1}(\boldsymbol{y}), \ \boldsymbol{V}^{i+1}(\boldsymbol{y}) \ldots \boldsymbol{V}^{I_1}(\boldsymbol{y})]$ is the payoff matrix of all securities other than those of firm i. The associated subspaces of $\mathbb{R}^S$ generated by the columns of the above matrices are denoted by $\mathcal{V}^0, \mathcal{V}^i(\boldsymbol{y}), \mathcal{V}(\boldsymbol{y})$ and $\mathcal{V}_{-i}(\boldsymbol{y})$ respectively: we call $\mathcal{V}^i(\boldsymbol{y})$ the *firm i-subspace* and $\mathcal{V}(\boldsymbol{y})$ the *marketed subspace* at $\boldsymbol{y}$.

A vector of prices $\boldsymbol{q}$ which prices the basic securities in the model (the columns of the matrix $\boldsymbol{V}(\boldsymbol{y})$) leads to a valuation of every income stream in the marketed subspace $v_q : \mathcal{V}(\boldsymbol{y}) \to \mathbb{R}$ defined by

$$v_q(\boldsymbol{m}) = q_0\xi_0 + \boldsymbol{q}_y\boldsymbol{\theta} + \sum_{i\in\mathcal{I}_1} \boldsymbol{q}_c^i\ \boldsymbol{\xi}_i$$

where $\boldsymbol{z} = (\xi_0, \boldsymbol{\theta}, (\boldsymbol{\xi}_i)_{i\in\mathcal{I}_1})$ is any portfolio such that $\boldsymbol{m} = \boldsymbol{V}(\boldsymbol{y})\boldsymbol{z}$. The valuation v_q is well-defined if the vector of prices $\boldsymbol{q}$ does not offer any arbitrage opportunities — a property which is equivalent to the existence of a strictly positive vector $\boldsymbol{\pi} \in \mathbb{R}^S$ such that $\boldsymbol{\pi}\boldsymbol{V}(\boldsymbol{y}) = \boldsymbol{q}$ (see Magill-Quinzii [1996, section 9]). Since we have assumed that there are investors ($\mathcal{I}_2 \neq \emptyset$) who can take advantage of arbitrage opportunities, any vector of equilibrium prices for the securities must be arbitrage free, and thus admit an associated vector of state prices.

Definition 2. A *financial market equilibrium with price perceptions* $\tilde{\boldsymbol{Q}}$ for the economy $\mathcal{E}(\boldsymbol{U}, \boldsymbol{\omega}_0, \boldsymbol{F}, \boldsymbol{\tau})$ is a triple

$$((\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{\theta}}, \bar{\boldsymbol{\xi}}), \bar{\boldsymbol{q}}\,; \tilde{\boldsymbol{Q}})$$

consisting of actions, prices and price perceptions such that

(i) for each agent $i \in \mathcal{I}$ the action $(\bar{\boldsymbol{x}}^i, \bar{e}^i, \bar{\kappa}^i, \bar{\boldsymbol{\theta}}^i, \bar{\boldsymbol{\xi}}^i)$ maximizes $U^i(\boldsymbol{x}^i, e^i)$ among consumption-effort streams such that

$$\begin{aligned} x_0^i &= \omega_0^i - v_{\bar{q}}(\boldsymbol{m}^i) + \tilde{\boldsymbol{Q}}_y^i(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)(1-\theta_i^i) - \tilde{\boldsymbol{Q}}_c^i(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)\boldsymbol{\xi}_i^i - \kappa^i \\ \boldsymbol{x}_{\boldsymbol{1}}^i &= \boldsymbol{m}^i + \boldsymbol{F}^i(\kappa^i, e^i)\theta_i^i + \boldsymbol{R}^i(\kappa^i, e^i)\boldsymbol{\xi}_i^i \end{aligned}$$

for $\kappa^i \in \mathbb{R}_+$ and $(\boldsymbol{\theta}^i, \boldsymbol{\xi}^i) \in \mathbb{R}^{I_1} \times \mathbb{R}^J$, with $\boldsymbol{m}^i = \boldsymbol{1}\,\xi_0^i + \sum_{k\neq i}(\bar{\boldsymbol{y}}^k\theta_k^i + \bar{\boldsymbol{R}}^k\boldsymbol{\xi}_k^i)$

(ii) $\bar{\boldsymbol{q}}^i = \tilde{\boldsymbol{Q}}^i(\bar{\kappa}^i, \bar{\boldsymbol{\theta}}^i, \bar{\boldsymbol{\xi}}^i), i \in \mathcal{I}_1$

(iii) $\sum_{i\in\mathcal{I}} \bar{\theta}_k^i = 1, k \in \mathcal{I}_1$ (iv) $\sum_{i\in\mathcal{I}} \bar{\xi}_0^i = 0$ (v) $\sum_{i\in\mathcal{I}} \bar{\boldsymbol{\xi}}_k^i = \boldsymbol{0},\ k \in \mathcal{I}_1$.

Note that this definition uses some obvious notation: $\bar{\boldsymbol{y}}^k = \boldsymbol{F}^k(\bar{\kappa}^k, \bar{e}^k)$ is the equilibrium output of firm k and $\bar{\boldsymbol{R}}_j^k$ is the payoff of the j^{th} option on firm k when its output is $\bar{\boldsymbol{y}}^k$.

In an equilibrium with price perceptions, each entrepreneur takes the production plans and the prices of the securities of other entrepreneurs' firms as given, correctly anticipating the effort they will invest in their firms. He chooses his own actions, anticipating that those which are observable (his financial decisions) will influence the prices of his securities in the way indicated by the function

$\tilde{\boldsymbol{Q}}^i(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$. By (ii), the price perceptions are consistent with the observed equilibrium prices $\bar{\boldsymbol{q}}^i$ for each firm, and by (iii)-(v) the security markets clear.

The price perception functions. Without more precise assumptions on the price perceptions $\tilde{\boldsymbol{Q}}^i$, the definition of equilibrium given so far only incorporates the first assumption that we discussed above — namely that investors have correct expectations — but it does not yet explicitly incorporate the second — namely that entrepreneurs are fully aware of this fact. To form his anticipations $\tilde{\boldsymbol{Q}}^i$, entrepreneur i needs to predict:

(a) the *output* of his firm that investors expect if they observe $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$

(b) how the market will *price* this expected output and the associated options of his firm.

For part (a) we use the assumption that entrepreneur i knows that investors will deduce from the observation of $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$ what his likely effort $e^i \in \tilde{e}^i$ will be, and hence what the likely output $\boldsymbol{F}^i(\kappa^i, e^i)$ of his firm will be. For part (b) we assume that the entrepreneur is, like an investor, a price-taker in the market for risky income streams. This price-taking assumption for price perceptions can be formalized as follows. If $\boldsymbol{m} \in \mathbb{R}^S$ is a potential income stream in $\mathcal{V}(\bar{\boldsymbol{y}})$, then its anticipated value is $v_{\bar{q}}(\boldsymbol{m}) = \sum_{s\in\mathcal{S}} \pi_s m_s$, where $\boldsymbol{\pi} \in \mathbb{R}^S_{++}$ is any vector of state prices satisfying $\boldsymbol{\pi} V(\bar{\boldsymbol{y}}) = \bar{\boldsymbol{q}}$. As long as the entrepreneur envisions alternative production plans lying in the marketed subspace $\mathcal{V}(\bar{\boldsymbol{y}})$, he does not perceive the possibility of affecting the state prices implicit in the equilibrium prices $\bar{\boldsymbol{q}}$. While the price-taking assumption leads to a well-defined valuation of income streams in the marketed subspace, it does not extend in any natural way to income streams outside of the marketed subspace: for if $\boldsymbol{m} \notin \mathcal{V}(\bar{\boldsymbol{y}})$, the value $\sum_{s\in\mathcal{S}} \pi_s m_s$ can change when the vector of state prices satisfying $\boldsymbol{\pi} V(\bar{\boldsymbol{y}}) = \bar{\boldsymbol{q}}$ is changed, so that the valuation of the stream $\boldsymbol{m}$ is no longer well-defined. To stay within a framework that permits the competitive assumption to be retained without raising conceptual difficulties, we introduce the assumption of partial spanning.

Definition 3. We say that there is *partial spanning* (PS) at $\bar{\boldsymbol{y}}$ if for all $i \in \mathcal{I}_1$, for all $(\kappa^i, e^i) \in \mathbb{R}^2_+$ and $\boldsymbol{y}^i = \boldsymbol{F}^i(\kappa^i, e)$, the firm i-subspace $\mathcal{V}^i(\boldsymbol{y})$ is contained in the marketed subspace at $\bar{\boldsymbol{y}}$, i.e. $\mathcal{V}^i(\boldsymbol{y}) \subset \mathcal{V}(\bar{\boldsymbol{y}})$.

The partial spanning assumption is classical—and is often used in the finance literature: it means that a firm cannot create a "new security", i.e. an income stream which is not in the existing marketed subspace $\mathcal{V}(\bar{\boldsymbol{y}})$, by changing its production plan. *With partial spanning the market prices of the securities are sufficient signals to value all possible alternative production plans of any firm and its associated options.*

Definition 4. A *financial market equilibrium with rational competitive price perceptions* (RCPP) is an equilibrium $((\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{\theta}}, \bar{\boldsymbol{\xi}}), \bar{\boldsymbol{q}}\,; \tilde{\boldsymbol{Q}})$ with price perceptions such that:

(i) PS holds at $\bar{\boldsymbol{y}}$

(ii) for each $i \in \mathcal{I}_1$ the price perceptions are given by

$$\tilde{\boldsymbol{Q}}_y^i(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i) = \bar{\boldsymbol{\pi}} \boldsymbol{F}^i(\kappa^i, \hat{e}^i), \quad \tilde{Q}_j^i(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i) = \bar{\boldsymbol{\pi}} \boldsymbol{R}_j^i(\kappa^i, \hat{e}^i),\ j \in \mathcal{J}^i$$

for any $\bar{\boldsymbol{\pi}} \in \mathbb{R}_{++}^S$ such that $\bar{\boldsymbol{\pi}} V(\bar{\boldsymbol{y}}) = \bar{\boldsymbol{q}}$ and $\hat{e}^i \in \tilde{e}^i(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$ which maximizes

$$\bar{\boldsymbol{\pi}} \boldsymbol{F}^i(\kappa^i, e^i)(1 - \theta_i^i) - \bar{\boldsymbol{\pi}} \boldsymbol{R}^i(\kappa^i, e^i) \boldsymbol{\xi}_i^i$$

Note that we use the notation $\tilde{e}^i(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$ instead of $\tilde{e}^i(\boldsymbol{m}^i, \kappa^i, \theta_i^i, \boldsymbol{\xi}_i^i)$, since $\boldsymbol{m}^i$ is a function of the financial variables $(\theta_k^i, \boldsymbol{\xi}_k^i, k \neq i)$ given by (4). To check if his equilibrium financial decisions $(\bar{\kappa}^i, \bar{\boldsymbol{\theta}}^i, \bar{\boldsymbol{\xi}}^i)$ are optimal, entrepreneur i considers alternative decisions $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$, recognizing that investors are rational and will deduce from $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$ what his associated optimal effort will be — namely the solution of the optimal effort problem (E) if it is unique, or if it is multivalued, the solution which yields the highest date 0 income for entrepreneur i (recall that $u_1^i(\boldsymbol{x}_{\mathbf{1}}^i) - c^i(e^i)$ has the same value for each of the solutions). This is the “rational” part of his anticipations. To evaluate the prices $\tilde{\boldsymbol{Q}}^i(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$ that he would get for the alternative output or that he would pay for the options on his firm, he uses any state price vector $\bar{\boldsymbol{\pi}}$ compatible with the equilibrium vector of security prices $\bar{\boldsymbol{q}}$. This is the “competitive” part of his expectations, which requires that PS hold at equilibrium.

PS is automatically satisfied if the financial markets are complete at equilibrium (rank $V(\bar{\boldsymbol{y}}) = S$), but it can also be satisfied when the markets are incomplete as shown by the following examples.

Example 1. The financial markets are simple: they consist solely of the bond and equity markets, so that $\mathcal{J}^i = \emptyset$ for all $i \in \mathcal{I}_1$. The production function of each firm has a factor structure: $\boldsymbol{F}^i(\kappa^i, e^i) = f^i(\kappa^i, e^i)\boldsymbol{\eta}^i$ where $f^i : \mathbb{R}_+^2 \longrightarrow \mathbb{R}$ is a concave increasing function and $\boldsymbol{\eta}^i \in \mathbb{R}_+^S$ is a fixed vector, characterizing the risk structure of the firm. Then PS is satisfied if $f^i(\bar{\kappa}^i, \bar{e}^i) > 0$ for all $i \in \mathcal{I}_1$. This case is studied in detail in Magill-Quinzii (1999).

Example 2. The financial securities consist of the riskless bond, equity and options on each firm. Suppose the uncertainty (shocks) affecting the production in the economy is decomposed into a product of $\mathcal{I}_1$ spaces

$$\mathcal{S} = \mathcal{S}^1 \times \ldots \times \mathcal{S}^{I_1} = \{1, \ldots, S^1\} \times \ldots \times \{1, \ldots, S^{I_1}\}$$

so that a state of nature is an I_1 -triple $s = (s^1, \ldots, s^{I_1})$ where s^i is the shock experienced by firm i. Then for any pair of states $s = (s^1, \ldots, s^{I_1}) \in \mathcal{S}, \hat{s} = (\hat{s}^1, \ldots, \hat{s}^{I_1}) \in \mathcal{S}$ with $s^i = \hat{s}^i$, $F^i_s(\kappa^i, e^i) = F^i_{\hat{s}}(\kappa^i, e^i)$ for all $(\kappa^i, e^i) \in \mathbb{R}^2_+$. If the vector $\boldsymbol{F}^i(\bar{\kappa}^i, \bar{e}^i)$ takes on S^i different values for the S^i individual states of firm i, and if there are options with striking prices in between the S^i different values taken by the output of firm i, for each firm $i \in \mathcal{I}_1$, then PS is satisfied.

3 Constrained Optimality of RCPP

The concept of an RCPP equilibrium is a natural way of describing market behavior in a production economy with moral hazard in which participants on the financial markets are well informed. To get a feel for how natural this concept is we turn to a study of its normative properties. As we mentioned earlier, at the first stage of development, when the contracts traded consist solely of the bond and the equity of firms, there is a clear trade-off between incentives and risk sharing. Entrepreneurs who want to finance their investment without incurring a large debt (which would put them in an in inordinately risky situation) can choose to finance some of their investment by issuing equity, thus opening the way to risk sharing and diversification. But issuing equity means they no longer receive the full marginal benefit of their effort, so their incentives to exert effort are reduced. Do markets induce entrepreneurs to make the optimal trade-off between incentives and risk sharing in their choice of debt and equity?

We also argued that, at a more mature stage of development, in addition to the bond and equity markets, options on the firms' profits (equity) are introduced. Such contracts not only augment the opportunities for risk sharing, but also permit the introduction of non-linear reward schedules for entrepreneurs: non-linear schedules incorporate "high powered" incentives which can help to solve the moral-hazard problem induced by the reduced equity shares of entrepreneurs. If the entrepreneur receives a larger share of output when the firm's realized output is high than when it is low, then he will (typically) be induced to increase effort, to increase the likelihood of a high realization of output. Such an incentive scheme can be obtained by adding options to his share of equity: but would an entrepreneur choose to buy options to increase his incentives in this way, given that the income stream received from his firm will tend to be more risky? In short, *do market-induced choices of bonds, equity and options by entrepreneurs and investors lead to the best possible use of these instruments*?

To answer this question we consider another way of arriving at an allocation where a "planner" — rather than the agents — chooses the financial variables, and examine if the planner could obtain a better allocation (in the Pareto sense) than that achieved in a RCPP equilibrium. Such a comparison only makes sense if the planner faces the same problem of unobservability of effort of the entrepreneurs and is restricted to the same opportunities for risk sharing as those available to the agents with the system of financial markets. In particular the planner cannot dictate effort levels to entrepreneurs — rather, these effort levels are chosen optimally by the entrepreneurs who take the reward structure given by the debt-equity-option choice of the planner and the effort levels of other agents (and hence their outputs) as given.

Definition 5. An allocation $(\boldsymbol{x}, \boldsymbol{e}) \in \mathbb{R}_+^{(S+1)I} \times \mathbb{R}_+^I$ is *constrained feasible* if there exist inputs and portfolios $(\boldsymbol{\kappa}, \boldsymbol{\theta}, \boldsymbol{\xi}) \in \mathbb{R}_+^I \times \mathbb{R}^{2I} \times \mathbb{R}^{IJ}$ such that

(i) $\sum_{i\in\mathcal{I}} \xi_0^i = 0$ (ii) $\sum_{i\in\mathcal{I}} \theta_k^i = 1,\ k \in \mathcal{I}$ (iii) $\sum_{i\in\mathcal{I}} \boldsymbol{\xi}_k^i = \mathbf{0},\ k \in \mathcal{I}$

(iv) $\sum_{i\in\mathcal{I}} x_0^i = \sum_{i\in\mathcal{I}} \omega_0^i - \sum_{i\in\mathcal{I}} \kappa^i$

(v) $\boldsymbol{x}_\mathbf{1}^i = \boldsymbol{m}^i + \boldsymbol{F}^i(\kappa^i, e^i)\theta_i^i + \boldsymbol{R}^i(\kappa^i, e^i)\boldsymbol{\xi}_i^i,\ i \in \mathcal{I}$

(vi) $\boldsymbol{m}^i = \mathbf{1}\xi_0^i + \sum_{k\neq i}(\boldsymbol{F}^k(\kappa^k, e^k)\theta_k^i + \boldsymbol{R}^k(\kappa^k, e^k)\boldsymbol{\xi}_k^i),\ i \in \mathcal{I}$

(vii) $e^i \in \tilde{e}^i(\boldsymbol{m}^i, \kappa^i, \theta_i^i, \boldsymbol{\xi}_i^i),\ i \in \mathcal{I}$

An allocation $(\boldsymbol{x}, \boldsymbol{e})$ is *constrained Pareto optimal* (CPO), if it is constrained-feasible, and if there does not exist any alternative constrained feasible allocation $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{e}})$ such that $U^i(\hat{\boldsymbol{x}}^i, \hat{e}^i) \geq U^i(\boldsymbol{x}^i, e^i)$, $i \in \mathcal{I}$, with strict inequality for at least one i.

(i)-(iii) are the feasibility constraints for the planner's choice of financial variables $(\boldsymbol{\theta}, \boldsymbol{\xi})$. Constraint (iv) indicates that the planner does not need to respect a system of prices for the securities and the associated date 0 budget constraint implied for each agent: it is in this sense that the "planner" replaces the "market". (v) and (vi) indicate that the planner's choice of date 1 consumption streams, and hence risk sharing, for the agents respects the existing structure of the financial securities. (vii) are the incentive constraints which reflect the fact that the choice of effort is made by entrepreneur i (and not the planner), and is the one that is optimal given the financial variables attributed to him, and given the effort levels of other agents (since agent i takes $\boldsymbol{m}^i$ as given).

Despite the fact that a planner chooses the financial variables $(\kappa, \boldsymbol{\theta}, \boldsymbol{\xi})$ fully aware of their consequences for the choices of effort by entrepreneurs and of the effect of each entrepreneur's effort on the consumption of other agents (the outside shareholders), he cannot improve on an RCPP equilibrium allocation arising from the self-interested choices of agents co-ordinated by the financial markets, provided we invoke the following strengthening of the partial spanning assumption.

Definition 6. We say that there is *strong partial spanning* (SPS) at $\bar{\boldsymbol{y}}$ if for all $(\kappa, e) \in \mathbb{R}^{2I_1}$ and $\boldsymbol{y} = (\boldsymbol{F}^k(\kappa^k, e^k)_{k\in\mathcal{I}_1})$, $\mathcal{V}_{-i}(\boldsymbol{y}) \subset \mathcal{V}_{-i}(\bar{\boldsymbol{y}})$ for all $i \in \mathcal{I}_1$.

SPS ensures that there is partial spanning for every subset of $I_1 - 1$ firms. Note that even if markets were complete, SPS would not automatically be satisfied. It holds if the securities based on the outputs of any subset of $I_1 - 1$ firms suffice to complete the markets, or if each firm spans its own subspace, as in Examples 1 and 2. SPS implies PS: if firm i cannot create an income stream which lies outside $\mathcal{V}^{-k}(\bar{\boldsymbol{y}})$ for $k \neq i$, it cannot create an income stream lying outside $\mathcal{V}(\bar{\boldsymbol{y}})$.

The need for the additional assumption SPS comes from the fact that in the RCPP equilibrium there are two potential sources of inefficiency. The first arises from the property that the equilibrium is a *Nash equilibrium*, in which the effort decision of each entrepreneur depends on the decisions of other entrepreneurs. The second arises from the *moral hazard problem*: the choice of effort of an entrepreneur affects the payoff of all the securities based on his firm and thus has an external effect on the investors who buy these securities. The assumption SPS cancels the possible inefficiency due to the Nash aspect of the concept. For the decisions of an entrepreneur depends on the decisions of the others only through the outside income possibilities offered by the securities based on their firms. Under SPS, a planner could not, by changing all the portfolios and capital choices of the entrepreneurs, create outside income possibilities which would induce a particular entrepreneur to a better effort decision, since the subspace of outside income has to be the same or a reduced version of the equilibrium subspace. In Magill-Quinzii (2000) we show that without the assumption SPS, an RCPP equilibrium can indeed be constrained inefficient, but that once SPS is invoked, an RCPP equilibrium is constrained efficient. The result of course depends on the fact that the model is a one-good, two-period finance model: without this assumption, i.e if there were more than two goods or more than two periods, the constrained inefficiency mentioned in the introduction—due to the feedback between agents' decisions and the spot prices—would reappear when security markets are incomplete.

Proposition B (RCPP is CPO). *If an RCPP equilibrium* $((\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{\theta}}, \bar{\boldsymbol{\xi}}), \bar{\boldsymbol{q}}\,; \tilde{\boldsymbol{Q}})$ *of the economy* $\mathcal{E}(\boldsymbol{U}, \boldsymbol{\omega}_0, \boldsymbol{F}, \boldsymbol{\tau})$ *satisfies SPS at* $\bar{\boldsymbol{y}}$*, then* $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{e}})$ *is constrained Pareto optimal.*

The choice of financial variables $(\boldsymbol{\theta}, \boldsymbol{\xi})$ creates a reward structure for each entrepreneur, namely a *contract* linking his payoff to the performance of his firm and the rest of the economy

$$\phi^i(y) = \theta^i_i y^i + \sum_{j\in\mathcal{J}^i} \xi^i_{i,j} \, \max\,\{y^i - \tau^i_j, 0\} + m^i(y^{-i}), \quad i \in \mathcal{I}_1$$

where $y = (y^i, y^{-i})$, y^i being the random output of the firm and y^{-i} the random output of all other firms. $\mathcal{J} = \bigcup_{i \in \mathcal{I}_1} \mathcal{J}^i$ determines the richness of the incentive structure over and above the basic equity contracts. If $\mathcal{J} = \emptyset$, the market and the planner are restricted to linear contracts, while if $\mathcal{J} \neq \emptyset$ the admissible contracts are nonlinear (piecewise linear): the larger the sets $\mathcal{J}^i$, the larger the admissible class of piecewise linear functions.

The CPO problem, which amounts to choosing optimally the investment, risk and incentive structure for the economy, is a generalized *principal-agent problem*, in which the planner (the principal) chooses the investment in each firm and the (constrained) optimal contract for each entrepreneur and investor in the economy. When agents rationally anticipate in the way described by an RCPP equilibrium, then Proposition B asserts that *a system of markets is capable of solving society's principal-agent problem.* The basic driving force for this optimality property of an RCPP equilibrium is that the social effect of each entrepreneur's choice of capital and reward structure — in particular the effect on outside investors — is transmitted to the entrepreneur through the rational price perceptions.

4 First-Order Conditions for CPO and RCPP

A way—indeed probably the best way—of understanding the forces which lead agents to optimally co-ordinate their actions is to study the first-order (i.e the marginal) conditions that must be satisfied at a CPO and to show how these end up being achieved at an RCPP equilibrium through the disciplining effect of the perception functions $\tilde{\boldsymbol{Q}}$.

First-order conditions for CPO. Let $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{e}, \boldsymbol{\kappa}, \boldsymbol{\theta}, \boldsymbol{\xi})$ be a CPO allocation such that: (i) the striking prices are strictly between the values $F^i_s(\kappa^i, e^i), s \in \mathcal{S}$, so that the payoffs $\boldsymbol{R}^i_j(\boldsymbol{y}^i)$ of the options are locally differentiable; (ii) each agent's consumption vector $\boldsymbol{x}^i$ is strictly positive; (iii) each entrepreneur i's effort level e^i is a locally differentiable selection of the effort correspondence $\tilde{e}^i$, which with a slight abuse of notation we denote by $\tilde{e}^i(\boldsymbol{m}^i, \kappa^i, \theta^i_i, \boldsymbol{\xi}^i_i)$. This CPO allocation is an extremum of the social welfare function

$$\sum_{i \in \mathcal{I}} \nu^i (u^i_0(x^i_0) + u^i_1(\boldsymbol{x}^i_{\boldsymbol{1}}) - c^i(e^i))$$

subject to the constraints (i)-(vii) in Definition 4, for some vector of relative weights $\boldsymbol{\nu} \in \mathbb{R}^I_{++}$. It must therefore satisfy the FOC for this constrained maximum problem. To express the cost of each constraint in units of date 0 consumption, we divide all the multipliers by the multiplier induced by the resource availability constraint (iv) at date 0. Let $(q_0, q^k_y, \boldsymbol{q}^k_c, 1, \boldsymbol{\pi}^i, \boldsymbol{\mu}^i, \epsilon^i)$ denote the resulting normalized multipliers associated with the constraints (i)-(vii). For each $s \in \mathcal{S}$ and $i \in \mathcal{I}$, let $\mathcal{J}^i_s \subset \mathcal{J}^i$ denote the subset of options which are "in the money" at the CPO in state s, i.e. $j \in \mathcal{J}^i_s$ implies $F^i_s(\kappa^i, e^i) > \tau^i_j$. The first-order conditions with respect to the variables $(\boldsymbol{x}^i, e^i, \boldsymbol{m}^i, \kappa^i, \boldsymbol{\xi}^i, \boldsymbol{\theta}^i)$ are:

$$\frac{\partial u^i_1/\partial x^i_s}{u^{i\prime}_0} = \pi^i_s, \quad s \in \mathcal{S} \tag{5}$$

$$\frac{c^{i\prime}}{u^{i\prime}_0} = \left[\sum_{s\in\mathcal{S}} \pi^i_s \left(\theta^i_i + \sum_{j\in\mathcal{J}^i_s} \xi^i_{i,j}\right) + \sum_{k\neq i}\sum_{s\in\mathcal{S}} \mu^k_s \left(\theta^k_i + \sum_{j\in\mathcal{J}^i_s} \xi^k_{i,j}\right)\right] \frac{\partial F^i_s}{\partial e^i} - \epsilon^i \tag{6}$$

$$\mu^i_s = \pi^i_s + \epsilon^i \frac{\partial \tilde{e}^i}{\partial m^i_s}, \quad s \in \mathcal{S} \tag{7}$$

$$1 = \left[\sum_{s\in\mathcal{S}} \pi^i_s \left(\theta^i_i + \sum_{j\in\mathcal{J}^i_s} \xi^i_{i,j}\right) + \sum_{k\neq i}\sum_{s\in\mathcal{S}} \mu^k_s \left(\theta^k_i + \sum_{j\in\mathcal{J}^i_s} \xi^k_{i,j}\right)\right] \frac{\partial F^i_s}{\partial \kappa^i} + \epsilon^i \frac{\partial \tilde{e}^i}{\partial \kappa^i} \tag{8}$$

$$q_0 = \sum_{s\in\mathcal{S}} \mu^i_s \tag{9}$$

$$q^k_j = \sum_{s\in\mathcal{S}} \mu^i_s R^k_{j,s} \tag{10}$$

$$q^k_y = \sum_{s\in\mathcal{S}} \mu^i_s F^k_s \tag{11}$$

$$q^i_j = \sum_{s\in\mathcal{S}} \pi^i_s R^i_{j,s} + \epsilon^i \frac{\partial \tilde{e}^i}{\partial \xi^i_{i,j}} \tag{12}$$

$$q^i_y = \sum_{s\in\mathcal{S}} \pi^i_s F^i_s + \epsilon^i \frac{\partial \tilde{e}^i}{\partial \theta^i_i} \tag{13}$$

To these equations should be added the FOC for the choice of optimal effort by entrepreneur i

$$\frac{c^{i\prime}(e^i)}{u^{i\prime}_0} = \sum_{s\in\mathcal{S}} \pi^i_s \left(\theta^i_i + \sum_{j\in\mathcal{J}^i_s} \xi^i_{i,j}\right) \frac{\partial F^i_s}{\partial e^i} \tag{14}$$

where we have divided both sides by $u^{i\prime}_0$. (14) is just the marginal way of expressing the incentive constraint $e^i = \tilde{e}^i(\cdot)$ in (vii). Using (6) and (14) gives

$$\epsilon^i = \sum_{k\neq i}\sum_{s\in\mathcal{S}} \mu^k_s \left(\theta^k_i + \sum_{j\in\mathcal{J}^i_s} \xi^k_{i,j}\right) \frac{\partial F^i_s}{\partial e^i} \tag{15}$$

Note that for an investor $\frac{\partial F^i_s}{\partial e^i} = 0$, $\frac{\partial F^i_s}{\partial \kappa^i} = 0$ which implies $\epsilon^i = 0$ and $\mu^i_s = \pi^i_s$.

Economic interpretation of FOC. Equation (5) defines the present-value vector $\pi^i = (\pi_1^i, \dots, \pi_S^i)$ of agent i: for any date 1 income stream $\boldsymbol{v} = (v_1, \dots, v_S)$, $\boldsymbol{\pi}^i \boldsymbol{v}$ is the present value to agent i of the income stream $\boldsymbol{v}$ (i.e. what he is prepared to pay for it at date 0). The components of the vector $\boldsymbol{q} = (q_0, q_y^k, \boldsymbol{q}_c^k, k \in \mathcal{I})$ are the shadow prices of the securities i.e. the social gain from giving one (marginal) unit of the relevant security to any agent in the economy. μ_s^i is the social gain from giving one more unit of income to entrepreneur i in state s: in most models this social gain would coincide with the private gain π_s^i, but in this setting, giving more income to entrepreneur i influences his effort, and thus has a consequence on other agents (equity or option holders of firm i), which creates a discrepancy between social and private benefit. ϵ^i is the social value of an additional unit of effort by entrepreneur i; by (6) ϵ^i is the difference between the social marginal benefit — namely the (marginal) benefit to entrepreneur i *plus* the benefit to every "outside investor" holding either the equity or options of firm i — and the social marginal cost, which here coincides with the private cost $c^{i\prime}/u_0^{i\prime}$, since entrepreneur i is the only one to bear the cost of his effort. Since effort is chosen optimally by entrepreneur i, by the "envelope theorem", or more precisely by the FOC (14), the welfare effect on the entrepreneur of a marginal change in his effort is zero. This explains why (6) and (14) lead to (15), namely *that the social value of an additional unit of effort by entrepreneur i is the value to agents other than himself of the additional output that his effort would create*. Note that the benefit to these agents $k \neq i$ is evaluated using μ_s^k rather than π_s^k and thus when k is an entrepreneur incorporates the incentive cost of giving him a marginal increment of income in state s. As soon as $\theta_i^k \neq 0$ or $\boldsymbol{\xi}_i^k \neq 0$ for some $k \neq i$, a marginal increment of effort by agent i has an *external effect* on agent k which is not taken into account when entrepreneur i makes his effort decision. ϵ^i, which is the cost of the incentive constraint (vii), is the sum of these external effects, and is in essence *the cost of separating the ownership and control of firm i*. This cost is explicitly taken into account by the planner when he chooses the financial variables $(\boldsymbol{\kappa}, \boldsymbol{\theta}, \boldsymbol{\xi})$.

Equations (9)-(13), i.e. the first-order conditions with respect to the financial variables $(\boldsymbol{\xi}, \boldsymbol{\theta})$, express the limited sense in which there must be equalization of marginal rates of substitution to achieve a CPO allocation, full equalization (in the general case) being prevented by the fact that income can only be distributed indirectly using securities, and that the incentive constraints of the agents must be satisfied. Equations (9)-(13) require that the social marginal cost of each security equal its social marginal benefit, the latter being a sum of two terms, one direct, the other indirect: the direct effect is the private benefit to an agent of the security's income stream, and the indirect effect is the social cost of the reduced effort made by agent i as a result of this increment to his income stream. For the outside variables $(\xi_0, \xi_{k,j}^i, \theta_k^i)$ the indirect effect is taken into account by

μ^i_s, for the inside variables $(\xi^i_{i,j}, \theta^i_i)$ it depends on the specific way in which the variable affects the entrepreneur's effort. The FOC (8) for the capital stock κ^i of firm i differs in that an increment to κ^i affects all agents holding one of the securities of firm i.

How the FOC for CPO are achieved at equilibrium. Since an RCPP equilibrium is constrained Pareto optimal, in such an equilibrium entrepreneurs must — just like the planner in a CPO problem — be induced to take into account the external effect of their effort on the welfare of others, namely the terms involving ϵ^i in equations (6)-(13). How is this effect transmitted to entrepreneurs?

The first point to note is that entrepreneur i raises money by selling a share $(1-\theta^i_i)$ of his equity and is thus concerned with the valuation $q^i_y = \bar{\boldsymbol{\pi}}\, \boldsymbol{y}^i$ that investors will assign to his firm. The assumption of competition implies that he doesn't perceive any effect of his actions on the vector of state prices $\bar{\boldsymbol{\pi}}$; the assumption of rationality implies that he perceives that the output $\boldsymbol{y}^i$ that investors anticipate from his firm is influenced by his choice of financial variables. Actually since the entrepreneur can typically raise the value of his equity by holding options—to convince investors that it is in his interest to make a high effort—the net proceeds from selling equity is $q^i_y(1-\theta^i_i) - \sum_{j \in \mathcal{J}^i} q^i_j \xi^i_{i,j}$, and it is this net value which is of concern to entrepreneur i. When he considers alternative financing decisions, he knows that investors will anticipate the output $\boldsymbol{y}^i = \boldsymbol{F}^i(\kappa^i, \tilde{e}^i(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i))$ and that this anticipation on their part will translate into the net proceeds for him $\bar{\boldsymbol{\pi}}\boldsymbol{y}^i(1-\theta^i_i) - \bar{\boldsymbol{\pi}}\boldsymbol{R}^i(\boldsymbol{y}^i)\boldsymbol{\xi}^{\boldsymbol{i}}_{\boldsymbol{i}}$ at date 0. It is his concern for the value of the equity that he sells, net of the cost of options, which leads the entrepreneur to take into account the interests of outside investors when he chooses his financial variables $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$. This can be seen by studying the first-order conditions for an entrepreneur's maximum problem in an RCPP equilibrium and comparing them with the FOC for a CPO allocation. Consider the maximum problem of agent i in Definition 2(i). Let $\boldsymbol{\lambda}^i = (\lambda^i_0, \lambda^i_1, \ldots, \lambda^i_S) \in \mathbb{R}^{S+1}_+$ denote the vector of multipliers induced by the $S+1$ budget constraints: the normalized vector

$$\bar{\boldsymbol{\pi}}^i = \frac{1}{\bar{\lambda}^i_0}(\bar{\lambda}^i_1, \ldots, \bar{\lambda}^i_S) = (\bar{\pi}^i_1, \ldots, \bar{\pi}^i_S)$$

is the present-value vector of agent i at the equilibrium. The FOC are

$$\frac{\partial u^i_1/\partial x^i_s}{u^{i\prime}_0} = \bar{\pi}^i_s, \quad s \in S \tag{16}$$

$$\frac{c^{i\prime}}{u^{i\prime}_0} = \sum_{s \in S} \bar{\pi}^i_s \left(\bar{\theta}^i_i + \sum_{j \in \mathcal{J}^i_s} \bar{\xi}^i_{i,j} \right) \frac{\partial F^i_s}{\partial e^i} \tag{17}$$

$$1 \;=\; \sum_{s\in\mathcal{S}} \bar{\pi}_s^i \left(\bar{\theta}_i^i + \sum_{j\in\mathcal{J}_s^i} \bar{\xi}_{i,j}^i\right) \frac{\partial F_s^i}{\partial \kappa^i} + \frac{\partial \tilde{Q}_y^i}{\partial \kappa^i}(1-\bar{\theta}_i^i) - \frac{\partial \tilde{\boldsymbol{Q}}_c^i}{\partial \kappa^i}\bar{\boldsymbol{\xi}}_i^i \tag{18}$$

$$\bar{q}_\alpha \;=\; \sum_{s\in\mathcal{S}} \bar{\pi}_s^i v_s^\alpha \;+\; \frac{\partial \tilde{Q}_y^i}{\partial z_\alpha^i}(1-\bar{\theta}_i^i) \;-\; \frac{\partial \tilde{\boldsymbol{Q}}_c^i}{\partial z_\alpha^i}\bar{\boldsymbol{\xi}}_i^i \tag{19}$$

where α is an index denoting any one of the traded securities [$\alpha = 0$ (bond) or $\alpha = k$ (equity of firm k) or $\alpha = (k,j)$ (option j of firm k)], $v^\alpha \in \mathbb{R}^S$ is its dividend stream, and z_α^i is the appropriate component of agent i's portfolio $\boldsymbol{z}^i = (\boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$. By substituting the expression for $\tilde{Q}^i$ given in Definition 4, one can show that the FOC (16)-(19) are the same as the FOC (5)-(13) for constrained Pareto optimality. Since this requires a certain amount of computation, for the convenience of the reader we spell out these calculations in the Appendix.

The intuition for the result is nevertheless clear from (18)-(19). By paying attention to the way investors in the securities of firm i react to his financial decisions $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$, through the partial derivatives $\left(\frac{\partial \tilde{Q}^i}{\partial \kappa^i}, \frac{\partial \tilde{Q}^i}{\partial z_\alpha^i}, \alpha = 0, \dots\right)$ of his perception function $\tilde{Q}^i$, entrepreneur i is led to take their interests into account. Thus with sophisticated participants, the capital markets ensure that self-interested behavior leads to a constrained socially optimal outcome.

5 Pareto Optimality: Can Capital Markets Mimic Arrow Securities?

The standard GEI model is fundamentally a model of intertemporal risk sharing, and it can be used to study how limitations of the instruments available for transferring income across time or sharing intertemporal risks affect the equilibrium outcome. In section 2 we have shown how incentives can be introduced into the model through the non-observability of the effort of the entrepreneurs who run (and are the original owners) of the firms. In sections 3 and 4 we have shown how the presence of rational and informed investors can force entrepreneurs to take into account the interests of the shareholders: this was made precise in Proposition B which asserts that the capital markets lead to an optimal trade-off between risk sharing and incentives, relative to the possibly incomplete structure of the markets.

In Section 2 we have shown that if we lived in an ideal world where states of nature are verifiable, then sole proprietorship and a complete set of Arrow securities would solve the moral hazard problem. However in the real world the complex array of business and technological shocks to which firms are subjected makes states of nature unverifiable, and hence a system based on Arrow securities unimplementable. To be enforceable in the courts of law, contracts must be based on events that are easily observable and verifiable by a third party: for a production economy of the kind considered in this paper this essentially means that the contracts must be based on the *observed outputs* of firms—precisely the property satisfied by the standard capital market instruments consisting of debt, equity, and options on equity. What is intriguing is that there is a way of showing that these instruments, which constitute society's response to the problem of enforceability, are able to collectively mimic the ideal system of Arrow securities. More precisely, under appropriate conditions, an RCPP equilibrium with capital markets gives the same Pareto optimal outcome as the equilibrium with Arrow securities—the SPA equilibrium of Section 2.2

The key property of an SPA equilibrium which leads to Pareto optimality is that each agent has a single budget constraint expressing equality of the present value his lifetime consumption expenditure and income: as a result each entrepreneur is led to maximize the present value of the profit of his firm, with a shadow price equal to his marginal cost for his effort. It can be shown (Magill-Quinzii (2000)) that the budget constraint of an RCPP equilibrium can be written in a form which mimics the budget constraint of an SPA equilibrium: each agent is subject to a budget constraint expressing the equality of the present value of his expenditure and of his income (including the full profit generated by his firm), but the incompleteness of the markets and the fact that the entrepreneur's effort must be credible, i.e optimal given his choice of financing, lead to two additional constraints: the first is a *spanning constraint* and the second an *incentive constraint*. We show that there is an abstract condition on the security structure, which we call the *spanning-overlap condition*, under which these latter constraints are not binding for agents at an equilibrium, so that *the RCPP equilibrium outcome coincides with the SPA equilibrium allocation*. The "spanning part" is what is required for optimal risk-sharing, namely that markets be *complete*—the usual condition for optimality in a standard risk-sharing GEI equilibrium. However complete markets is not enough to deal with incentives: some extra "control" is needed so that an entrepreneur can simultaneously choose his risk and retain appropriate incentives.

To formalize this controllability condition, note that for each firm i, the securities can be placed into two categories: those whose payoffs are independent of the effort of entrepreneur i (e.g the equity and options of other firms, or the default-free bond) and those whose payoff is directly affected by his effort (e.g his equity or options on his equity). The overlap condition requires that there is an intersection (of dimension at least one) between the subspace spanned by the i-dependent and i-independent securities, for each entrepreneur i in the economy. When the spanning-overlap condition is satisfied, by adjusting the component of an entrepreneur's future income on the i-dependent securities the magnitude of the incentive effect can be adjusted, while the component on the i-independent subspace permits the risk profile to be kept at any desired level. Thus short risk sharing and incentives can be completely controlled.

Does this spanning-overlap condition have any chance of holding in the "real world"? In Magill-Quinzii (2000) we show that there are reasonable conditions on the structure of uncertainty that affects the firms, such that the condition can be satisfied by standard capital markets instruments. In order that output-contingent securities can do the work expected of state-contingent securities, the states must be distinguishable by the firms' outputs: two distinct shocks must always lead to different outcomes, for at least *some* firm in the economy. Then, if enough options are introduced on the equity of each firm (and recall that introducing such options is essentially costless), and if these options have "appropriate" striking prices, then a security structure consisting of debt, equity and options satisfies the spanning-overlap condition. Equity provides the basic output-contingent security for each firm: the options, contingent on the payoff of the equity, then provide a rich enough family of instruments so that not only can risks be shared, but also an appropriate non-linear incentive schedule can be created for each entrepreneur to induce him to make the appropriate effort. Thus the classical markets instruments consisting of debt, equity and options, when used by sophisticated and well-informed investors, can replace the ideal Arrow securities which are not observed in the real world. In this way the capital market instruments characteristic of developed financial markets provide a class of indirect instruments which lead us back to Adam Smith's invisible hand, co-ordinating the self-interested actions of agents, even in the presence of moral hazard.

Appendix

Let us show that when the expression for rational, competitive price perceptions $\tilde{\boldsymbol{Q}}^i$ given by Definition 4 is substituted in (18, 19) the FOC of an RCPP equilibrium coincide with the FOC for constrained Pareto optimality. The partial derivatives of $\tilde{\boldsymbol{Q}}^i$ are

$$\frac{\partial \tilde{Q}^i_\beta}{\partial \kappa^i} = \sum_{x \in \mathcal{S}^i_\beta} \bar{\pi}_s \left(\frac{\partial F^i_s}{\partial \kappa^i} + \frac{\partial F^i_s}{\partial e^i} \frac{\partial \tilde{e}^i}{\partial \kappa^i} \right) \tag{20}$$

$$\frac{\partial \tilde{Q}^i_\beta}{\partial \theta^i_k} = \sum_{s \in \mathcal{S}^i_\beta} \bar{\pi}_s \frac{\partial F^i_s}{\partial e^i} \left(\sum_{s' \in \mathcal{S}} \frac{\partial \tilde{e}^i}{\partial m^i_{s'}} y^k_{s'} \right), \ k \neq i \tag{21}$$

$$\frac{\partial \tilde{Q}^i_\beta}{\partial \xi^i_{k,j}} = \sum_{s^i \mathcal{S}^i_\beta} \bar{\pi}_s \frac{\partial F^i_s}{\partial e^i} \left(\sum_{s' \in \mathcal{S}^k_j} \frac{\partial \tilde{e}^i}{\partial m^i_{s'}} (y^k_{s'} - \tau^k_j) \right), \; k \neq i, \quad j \in \mathcal{J}^k \tag{22}$$

$$\frac{\partial \tilde{Q}^i_\beta}{\partial \theta^i_i} = \sum_{s^i \mathcal{S}^i_\beta} \bar{\pi}_s \frac{\partial F^i_s}{\partial e^i} \frac{\partial \tilde{e}^i}{\partial \theta^i_i} \tag{23}$$

$$\frac{\partial \tilde{Q}^i_\beta}{\partial \xi^i_{i,j}} = \sum_{s \in \mathcal{S}^i_\beta} \bar{\pi}_s \frac{\partial F^i_s}{\partial e^i} \frac{\partial \tilde{e}^i}{\partial \xi^i_{i,j}} \tag{24}$$

where β is an index denoting one of the securities associated with firm i ($\beta = y$ or $\beta = j$, $j \in \mathcal{J}^i$), whose price is influenced by the action of entrepreneur i, and $\mathcal{S}^i_\beta$ is the subset of states in which security (i, β) has a positive payoff: thus $\mathcal{S}^i_\beta = \mathcal{S}$ if $\beta = y$ and $\mathcal{S}^i_\beta = \mathcal{S}^i_j = \{s \in \mathcal{S} \mid F^i_s(\bar{\kappa}^i, \bar{e}^i) > \tau^i_j\}$ if $\beta = j, \; j \in \mathcal{J}^i$.

For $i \in \mathcal{I}$, define

$$\epsilon^i = (1 - \bar{\theta}^i_i) \sum_{s \in \mathcal{S}} \bar{\pi}_s \frac{\partial F^i_s}{\partial e^i} - \sum_{j \in \mathcal{J}^i} \bar{\xi}^i_{i,j} \sum_{s \in \mathcal{S}^i_j} \bar{\pi}_s \frac{\partial F^i_s}{\partial e^i} \tag{25}$$

$$\bar{\mu}^i_{s'} = \bar{\pi}^i_{s'} + \epsilon^i \frac{\partial \tilde{e}^i}{\partial m^i_{s'}}, \quad s' \in \mathcal{S} \tag{26}$$

Substituting (21) and (22) into equation (19) for a security α whose payoff is not directly influenced by the effort of agent i ($\alpha = (k, \beta)$ with $k \neq i$, $\beta = y$ or $\beta = j, j \in \mathcal{J}^k$) and using the expressions (25) and (26), we obtain

$$\bar{q}^k_\alpha = \sum_{s \in \mathcal{S}} \bar{\mu}^i_s v^\alpha_s$$

so that each agent equalizes the price of a security which influences his outside income with its present value under the modified present-value vector (26). Thus for an agent $k \neq i$ and a security for firm i ($\alpha = (i, \beta)$)

$$\bar{q}_\alpha = \bar{q}^i_\beta = \sum_{s \in \mathcal{S}} \bar{\mu}^k_s v^\alpha_s = \sum_{s \in \mathcal{S}} \bar{\pi}_s v^\alpha_s \tag{27}$$

where the second equality follows from the definition of $\bar{\boldsymbol{\pi}}$. Thus the valuations under the vectors $\bar{\boldsymbol{\pi}}$ and $\bar{\boldsymbol{\mu}}^k$ agree on the subspace $\mathcal{V}_{-k}(\bar{\boldsymbol{y}})$. A marginal change $\Delta\kappa^i$ in the input, or Δe^i in the effort of entrepreneur i, induces a change $\Delta y^i_s = \frac{\partial F^i_s}{\partial \kappa^i} \Delta\kappa^i$ or $\frac{\partial F^i_s}{\partial e^i} \Delta e^i$ in output in each state: this induces a change $\Delta \boldsymbol{y}^i$ in the payoff of equity and a change $\Delta \boldsymbol{R}^i_j$ in the payoff of option j where

$$\Delta R^i_{js} = \begin{cases} \frac{\partial F^i_s}{\partial \kappa^i} \; \Delta\kappa^i \text{ or } \frac{\partial F^i_s}{\partial e^i} \Delta e^i & \text{if} \quad s \in \mathcal{S}^i_j \\ 0 & \text{if} \quad s \notin \mathcal{S}^i_j \end{cases}$$

By SPS the changes $\Delta \boldsymbol{y}^i$ and $\Delta \boldsymbol{R}^i_j$ must lie in $\mathcal{V}_{-k}(\bar{\boldsymbol{y}})$ for all $k \neq i$. In view of (27)

$$\sum_{s\in\mathcal{S}^i_\beta} \bar{\pi}_s \frac{\partial F^i_s}{\partial \kappa^i} \;=\; \sum_{s\in\mathcal{S}^i_\beta} \bar{\mu}^k_s \frac{\partial F^i_s}{\partial \kappa^i}, \quad \sum_{s\in\mathcal{S}^i_\beta} \bar{\pi}_s \frac{\partial F^i_s}{\partial e^i} \;=\sum_{s\in\mathcal{S}^i_\beta} \bar{\mu}^k_s \frac{\partial F^i_s}{\partial e^i} \tag{28}$$

where we recall that $\mathcal{S}^i_\beta = \mathcal{S}$ when $\beta = y$. Since

$$1 - \bar{\theta}^i_i \;=\; \sum_{k\neq i} \bar{\theta}^i_k, \quad \bar{\xi}^i_{i,j} = -\sum_{k\neq i} \bar{\xi}^k_{i,j} \tag{29}$$

using (28), ϵ^i in (25) can be written as

$$\begin{aligned} \epsilon^i \;&=\; \sum_{k\neq i} \bar{\theta}^k_i \sum_{s\in\mathcal{S}} \bar{\mu}^k_s \frac{\partial F^i_s}{\partial e^i} \;+\; \sum_{k\neq i} \sum_{j\in\mathcal{J}^i} \bar{\xi}^k_{i,j} \sum_{s\in\mathcal{S}^i_j} \bar{\mu}^k_s \frac{\partial F^i_s}{\partial e^i} \\ &=\; \sum_{k\neq i} \sum_{s\in\mathcal{S}} \bar{\mu}^k_s \left(\bar{\theta}^k_i \;+\; \sum_{j\in\mathcal{J}^i_s} \bar{\xi}^k_{i,j} \right) \frac{\partial F^i_s}{\partial e^i} \end{aligned} \tag{30}$$

which is the same as (15). Substituting (20)-(24) into (17)-(19), using (28) and (30), gives the FOC (6)-(13) for a CPO.

It is interesting to note that when ϵ^i is defined by (25), and price perceptions satisfy (20)-(24), then for any change dz^i_α in the portfolio of entrepreneur i

$$\frac{\partial}{\partial z^i_\alpha} \left(\tilde{Q}^i_y (1 - \bar{\theta}^i_i) - \tilde{\boldsymbol{Q}}^i_c \bar{\boldsymbol{\xi}}^i_i \right) = \epsilon^i \frac{\partial \tilde{e}^i}{\partial z^i_\alpha}$$

Thus in an RCPP equilibrium *an entrepreneur acting purely in his own self interest is made aware of the value of his effort* (ϵ^i) *through the change in the date 0 income earned from the sale of his equity (net of options)*, arising from a change Δe^i in the effort that investors expect from him. The optimality property of an RCPP equilibrium is then explained by equality (30): market clearing and the common valuation of the traded securities imply that *the private value* ϵ^i *of his effort to entrepreneur* i given by (25) coincides with the social value of his effort to investors holding securities of firm i, given by the right side of (30).

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