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THE DYNAMICS OF CAPACITY ADJUSTMENTS IN A COMPETITIVE ECONOMY

J. Jaskold GABSZEWICZ and M. QUINZII*

C.O.R.E., 1348 Louvain-la-Neuve, Belgium

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In this paper we consider a simple model of a competitive economy, focusing our analysis on the problem of capacity adjustments resulting from a time-lag between production decisions and the availability of investments. Our main proposition shows that a particular form of myopic behaviour of the productive sector about output and price anticipations combined with instantaneous price adjustments, leads the economy to the efficient steady state where capacity is optimally adapted to the repeated level of economic activity. By contrast, persistent wage inflexibility at a too high level generates a permanent decline of capacity.

1. Introduction

This paper is an attempt to explore the dynamics of a simple competitive model when the effects of capacity adjustments operate with a time-lag of one period on current production decisions. The existence of this time-lag stems from the evidence that today-production is constrained to be realized with the amount of capital inherited from the past, even if the operating capital–labour ratio is ill-adapted to current factor prices. Today's decisions affecting the amount of capacity reveal their effects in the future only, while variable factors like labour can be freely adjusted in the short run. In fact, it is the very same evidence which is at the root of the usual microeconomic distinction between fixed and variable factors, and thus we borrow directly from this distinction the basic device which explains the adjustment of economic variables over time. To introduce the reader to the dynamic adjustment considered in this paper, we present it first in the following partial equilibrium framework.

Consider an economy whose productive sector is described by an aggregate production function $Y=f(K,L)$. Suppose that the prices of capital and labour are constant through time and that the capital available in period t has been chosen in period $t-1$ and cannot be modified in period t, the

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length of which is chosen precisely equal to the economic life of installed capacity. If K_{t-1} denotes this quantity of capital, inherited from period $t-1$ and active in period t, the short-run cost function in period t is a function $C_{K_{t-1}}(Y)$ obtained by using the minimum quantity of labour, given K_{t-1}, to produce Y.

The long-run cost function $C(Y)$ corresponds to the optimal choice of both capital and labour, given the factor prices, to produce a quantity Y of output.

Suppose that, at each period, the output market is competitive, so that the quantity sold on the market by the productive sector is such that its (short-run) marginal cost equals its price. We can easily represent the equilibrium on this market at period t using a partial analysis diagram (see fig. 1).

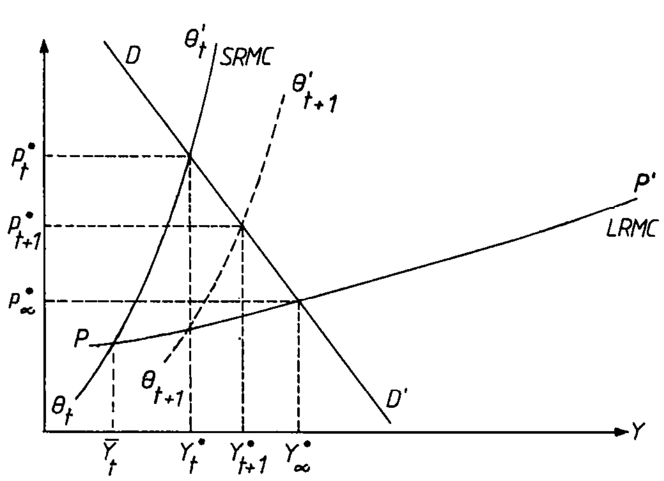

Fig. 1

In fig. 1, DD' represents the demand curve for the good, which is assumed to be the same at each period. $\theta_t \theta'_t$ is the supply curve in period t which coincides, since the market is competitive, with the short-run marginal cost curve of the productive sector endowed with the capital K_{t-1}; (Y^*_t, P^*_t) is the equilibrium of period t.

We also represent in the same figure the long-run marginal cost curve PP', which reveals that the existing level of capital K_{t-1} is ill-adapted to produce Y^*_t.

Let us call *capacity* of the productive sector corresponding to a level of capital $\bar{K}$, that level of output $\bar{Y}$ for which the short-run total cost equals the long-run total cost. The capacity $\bar{Y}$ is the level of output for which $\bar{K}$ is optimal, i.e.,

$$C_{\bar{K}}(\bar{Y}) = \min_K C_K(\bar{Y}) = C(\bar{Y}).$$

It follows easily from the envelope theorem that $C'_{\bar{K}}(\bar{Y}) = C'(\bar{Y})$; at $\bar{Y}$, the long-run and the short-run marginal cost are equal. Therefore, in fig. 1 $\bar{Y}_t$ corresponds to the capacity in period t of the productive sector, which is clearly undercapacitated to produce Y_t^*.

Accordingly, if we suppose that the expectations of the firms are naïve with respect to the observed output, a new equipment will be built at the end of period t to adapt the capacity to the quantity Y_t^*.[1] This choice of K_t leads in turn to a new short-run marginal cost curve for period $t+1$, depicted by the dotted line $\theta_{t+1}\theta'_{t+1}$, and to the equilibrium values (Y_{t+1}^*, P_{t+1}^*). Again the productive sector will observe in period $t+1$ that K_t is not well adapted to produce Y_{t+1}^* and will adjust the capacity of the next period to Y_{t+1}^*. Eventually, the adjustment will lead to the steady state (Y_∞^*, P_∞^*) where the installed equipment, price and output are identically renewed from period to period.

In spite of its simplicity the adjustment process we have just described has the merit of incorporating an important ingredient of the observed path of firms' behaviour. If the current capital–labour ratio is underadapted to the observed level of productive activity, there is a clear incentive to increase capacity so as to reduce productive costs, and conversely in case of overcapacity. The partial equilibrium approach exhibits however two major shortcomings. First, in their endeavour to adjust optimally the level of the productive factors to the observed output level, firms modify in each period their demand for these productive factors and, accordingly, factor prices may well fluctuate if they adjust instantaneously at their equilibrium values. Furthermore, since factor prices also determine the income distribution between the factor owners, such fluctuations may in turn induce fluctuations in the supply of capital, to the extent that the rate of savings should depend on the resulting income distribution. Since partial analysis must assume *constant* factor prices, it cannot reverberate all the effects of this adjustment process, which clearly needs a general equilibrium framework.

In the next section, we provide a simple general equilibrium framework in which the dynamic behaviour heuristically described above can be formally explored. At the end of this exploration we shall conclude that, under reasonable conditions, one should not expect 'pathological' capital trajec-

[1]This process differs from the classical 'Cobweb' model where firms adjust their supply according to naïve expectations about the selling price: the price at period $t+1$ is anticipated to remain equal to the price at period t. Here firms assume that the *quantity* produced will remain the same at period $t+1$, and adjust accordingly their amount of capital. The latter process is easily seen to stabilize itself much easier than the former.

tories if factor prices adjust instantaneously at their equilibrium values: the economy must converge to a steady state which is efficient in the sense that the capital–labour ratio is optimally adjusted to the level of economic activity. Then, in section 3, we move to a related question which has some interest in the present revival of Disequilibrium Economics: how wage rigidities could distort the process if adjustments take place on the labour market according to quantity, rather than price? We shall show that if the wage rate remains fixed at a 'too' high level, the whole economy may well be driven down for ever on a destructive path; a conclusion also obtained in a different framework by Ito (1979) and Picard (1979). Finally, our assumptions are briefly discussed in a concluding section. All proofs are given in an appendix.

2. The behaviour of the system with instantaneous price adjustments

In order to keep the model as simple as possible, we assume that there is only one composite commodity Y, a single type of labour L, and a single type of capital K, consisting of the commodity produced. The time subscript t refers to a given finite interval of time which is assumed equal to the economic life of the capacity installed at the end of period $t-1$.[2] The *consumption sector* divides into *labour owners* and *capital owners*. At period t, labour owners sell their labour l_t to the productive sector at a wage rate w_t and consume a quantity C_t^1 of the composite commodity bought at price p_t. In the sequel, we take the composite commodity as the numeraire, and set $p_t = 1$ for all t. We assume that the total labour force is equal to $\bar{L}$, and constant over time. The saving activity is performed by the capital owners. At the beginning of period t, firms reimburse to these capital owners an amount of product equal to $S_{t-1}(1+\rho_{t-1}^*)$; S_{t-1} represents the amount of capital lent to the productive sector at period $t-1$, and ρ_{t-1}^* the interest rate which prevailed at the same period. Furthermore capital owners are also assumed to own the productive sector so that the period-t profits

$$\Pi_t \underset{\text{def}}{=} Y_t - w_t L_t - (1+\rho_{t-1}^*) S_{t-1}$$

are also transferred to them. Capital owners then decide how to allocate their current income $R_t \underset{\text{def}}{=} (1+\rho_{t-1}^*) S_{t-1} + \Pi_t$ between present consumption C_t^2 and saving S_t.

We make the following assumptions about the consumption sector:

[2]Thus we assume that the quantity of equipment K_{t-1} decided at the end of period $t-1$, starts to become 'effective' at the beginning of period t, but cannot be changed inside the period. By assumption, it ceases to be effective at the end of period t. The quantity of capital K_t, decided at the end of period t, thus combines the *replacement* of K_{t-1} and the new *investment* (or disinvestment) possibly decided in the same period.

(A.1) $l_t(w_t)=\bar{L}$; the supply of labour is independent of wages and equal to the labour force $\bar{L}$.

(A.2) $S_t=\alpha(\rho_t)\cdot R_t$, $0<\alpha(\rho_t)<1$; $\alpha'(\rho)>0$;[3] $\alpha(\rho)\to 0$ if $\rho\to 0$ and $\alpha(\rho)\to 1$ if $\rho\to +\infty$; the savings of capital owners are proportional to their real income; furthermore, the proportion of income devoted to saving is an increasing function of the interest rate ρ_t.

The productive sector has production possibilities described by a Cobb–Douglas function F with constant returns to scale. Taking into account our assumption that the level of capital 'effective' in period t has been decided in period $t-1$, the production possibilities available in period t are

$$Y_t=F(K_{t-1},L)=AK_{t-1}^{\gamma}L^{1-\gamma}. \tag{1}$$

For simplicity we take $A=1$ in the sequel.

To adapt the dynamics considered in our introductory section to the model we have just described, we proceed in the following manner. At period t, both the supply of the composite commodity Y_t and the demand L_t for labour are derived from short-run profit maximization at given w_t and K_{t-1}. At the end of period t, the production sector has to decide the amount of capital K_t to be in use at period $t+1$. *We assume that K_t is chosen so as to minimize the cost of producing the observed output level Y_t, given the prices w_t and ρ_t.* Finally prices are assumed to clear instantaneously all markets at period t. Let us call the resulting state of the economy a temporary equilibrium. Formally, a *temporary equilibrium* is a set of values $\{Y_t\ K_t,\ L_t,\ C_t^1,\ C_t^2,\ w_t^*,\ \rho_t^*\}$ such that

(i) Y_t and L_t solve

$$\max_{(Y,L)} Y-w_t^*L-(1+\rho_{t-1}^*)K_{t-1} \quad \text{s.t.} \quad Y=F(K_{t-1},L),$$

(ii) K_t solves

$$\min_{(K,L)} w_t^*L+(1+\rho_t^*)K \qquad \text{s.t.} \quad F(K,L)=Y_t,$$

(iii) $K_t=S_t$, $L_t=\bar{L}$, $Y_t=C_t^1+C_t^2+S_t$.

[3]It is interesting to notice that the assumption $\alpha'(\rho)>0$ excludes the case of constant marginal propensity to save ($\alpha'(\rho)=0$). We have however verified that the whole analysis carries over in a much simpler manner under the assumption of constant propensity to save. Moreover in that case our main proposition extends to the whole class of constant returns to scale production functions satisfying the so-called 'Inada conditions' [cf. Jones (1975, p. 75)]. It is in fact the dependence of the propensity to save on the interest rate which makes the proof of our proposition much more difficult, but also more interesting!

In words a temporary equilibrium in period t is a state of the economy where (i) the productive sector maximizes its profits at given prices, conditional on the amount of capital inherited from period $t-1$, (ii) the amount of capital decided in period t minimizes the cost of producing the observed output level Y_t at current prices, and (iii) supply equals demand on each market. Implicit in this definition is the assumption that the productive sector in period t behaves about factor prices expectations as it behaves concerning expected future production: both are anticipated to remain the same in the next period. This is the simplest way to capture the idea we have put forward in our introduction: namely the incentive of firms to increase capital equipment when the capital–labour ratio is underadapted to current production, and to decrease it in the opposite case.

Let us compute the equilibrium values, in period t, as a function of the capital K_{t-1} inherited at the beginning of period t.

(a) Equilibrium on the labour market

Short-run profit maximization of the productive sector yields a demand for labour L_t such that the marginal productivity of labour equals the wage rate w_t^*, i.e.,

$$w_t^* = (1-\gamma)K_{t-1}^{\gamma}L_t^{\gamma}. \tag{2}$$

Since, from (A.1), the supply of labour is inelastic and equal to $\bar{L}$, the equilibrium equations on the labour market are

$$L_t = \bar{L}, \tag{3}$$

$$w_t^* = (1-\gamma)K_{t-1}^{\gamma}\bar{L}^{\gamma}. \tag{4}$$

(b) Equilibrium on the capital market

The production of period t is

$$Y_t = K_{t-1}^{\gamma}\bar{L}^{1-\gamma}. \tag{5}$$

The profit of the production sector is

$$\begin{aligned}\Pi_t &= Y_t^* - w_t^*\bar{L} - (1+\rho_{t-1}^*)K_{t-1}^* \\ &= \gamma K_{t-1}^{\gamma}\bar{L}^{1-\gamma} - (1+\rho_{t-1}^*)K_{t-1}^*.\end{aligned}$$

The revenue of the capital owners is

$$R_t = \Pi_t + (1+\rho_{t-1}^*)K_{t-1}^*,$$

$$R_t = \gamma K_{t-1}^{\gamma} \bar{L}^{1-\gamma}.$$

From Assumption (A.2), the supply of capital is then

$$S_t = \alpha(\rho_t^*)\gamma K_{t-1}^{\gamma} \bar{L}^{1-\gamma}. \tag{6}$$

According to the adjustment process described above we derive the demand for capital at period t as the result of the minimization problem (P):

$$\min_{K,L} w_t^* L + (1+\rho_t)K \quad \text{s.t.} \quad K^{\gamma} L^{1-\gamma} = Y_t. \tag{P}$$

Denote by L_{t+1}^e and K_t the solutions to this minimization problem. From the first-order condition we obtain the demand for capital K_t, i.e.,

$$K_t = \left(\left(\frac{\gamma}{1-\gamma}\right)^{1-\gamma} w_t^{*\,1-\gamma} \Big/ (1+\rho_t)^{1-\gamma} \right) Y_t. \tag{7}$$

Substituting (3) and (4) in this expression we get

$$K_t = (\gamma^{1-\gamma} \bar{L}^{(1-\gamma)^2} / (1+\rho_t)^{1-\gamma}) K_{t-1}^{\gamma(2-\gamma)}, \tag{8}$$

an equation which expresses the demand for capital at period t as a function of the inherited stocks K_{t-1} and the interest rate ρ_t. It remains to compute the equilibrium interest rate ρ_t^*. This is obtained by the clearing condition $S_t = K_t$. Combining (6) and (8) we get

$$\alpha(\rho_t)(1+\rho_t)^{1-\gamma} = \frac{1}{\gamma^{\gamma}} \cdot \frac{K_{t-1}^{\gamma(1-\gamma)}}{\bar{L}^{\gamma(1-\gamma)}}. \tag{9}$$

By Assumption (A.2), the left-hand term of (9) is a strictly increasing function of ρ_t which tends to 0 when $\rho_t \to 0$ and to $+\infty$ when ρ_t tends to $+\infty$. Thus eq. (9) defines without ambiguity the equilibrium interest rate $\rho_t^*(K_{t-1})$, which yields finally the equation

$$K_t = \alpha(\rho_t^*(K_{t-1}))\gamma K_{t-1}^{\gamma} \bar{L}^{1-\gamma}. \tag{10}$$

Eq. (10) is the basic difference equation which describes the evolution of capital over time.

(c) Equilibrium on the market for the consumption good

The demand for the product by the labour owners is

$$C_t^1 = w_t^* \bar{L} = (1-\gamma) K_{t-1}^{\gamma} \bar{L}^{1-\gamma}. \tag{11}$$

The demand by the capital owners is

$$C_t^2 = [1-\alpha(\rho_{t-1}^*)]\gamma K_{t-1}^{\gamma} \bar{L}^{1-\gamma}. \tag{12}$$

Equilibrium on the labour market and the capital market implies, by Walras Law, equilibrium on the good market. It is easy to check that

$$C_t^1 + C_t^2 + S_t = Y_t.$$

At this point, some remarks are in order. First notice that the equilibrium interest rate ρ_t^* is not, in general, equal to the marginal productivity r_{t+1} of capital K_t made available for period $t+1$. The actual ex-post rate of return of K_t in period $t+1$ is

$$1 + r_{t+1} = \gamma K_t^{\gamma-1} \bar{L}^{1-\gamma},$$

[recall that at period $t+1$, the labour market is in equilibrium at $\bar{L}$ and that $Y_{t+1} = K_t^{\gamma} \bar{L}^{1-\gamma}$ [see (5)]]. On the other hand, since the values K_t and L_{t+1}^e are solutions of program (P), it must be that

$$1 + \rho_t^* = \gamma K_t^{\gamma-1} (L_{t+1}^e)^{1-\gamma},$$

and that

$$K_t^{\gamma} (L_{t+1}^e)^{1-\gamma} = Y_t.$$

Consequently, as long as $Y_{t+1} \neq Y_t$ (and we will see in Proposition 1 that this is indeed the case except at the steady state), it must be that $L_{t+1}^e \neq \bar{L}$, and thus $\rho_t^* \neq r_{t+1}$.

Eq. (10), describing the time path of the capital stock is akin of the equation which arises in the simpler model of growth theory, where the stock of capital in period $t+1$ is determined by the production of period t and the propensity to save of the capitalist sector. However the difference is noticeable. Instead of assuming a fixed propensity to save which determines mechanically the evolution of capital and then supposes that the interest rate adjusts to the marginal productivity of capital, we introduce explicitly the working forces of the capital market. Both demand and supply depend on the interest rate ρ_t. This assumption is consistent with the length of the period that we have in mind: period t is long enough as to entail full depreciation of the installed capacity at period $t-1$. It is well-known that in the short run, fluctuations of the interest rate have little influence on the saving activity; in the period we consider however, one must expect that consumption and saving habits can be significantly affected by the level of the interest rate. We have translated this dependence by assuming that the total revenue R_t – which can be viewed as the wealth accumulated by capitalists over period t – is split between consumption and savings in a fraction which explicitly depends on the interest rate. This interest rate, except in the eventual steady state, does not equal the marginal productivity of capital in period $t+1$ since the expectations of the firm concerning prices and quantities are not fulfilled.

It can be objected to the way the demand for capital is derived that, in a competitive model, the firms should choose capital at the end of period t in order to maximize their expected profit in period $t+1$. But, with constant returns to scale, profit maximization is not sufficient to determine demand. The level of production must be forecasted. We have retained an assumption of complete naïve expectations concerning both quantities and prices. To base expectations on currently observed magnitudes is classical in macro-economics. However since anticipations concern a rather long period of time, it would perhaps have been more reasonable to let expectations reflect subjective anticipations of entrepreneurs on the evolution of the economy. For instance we could have written the expected level Y^e_{t+1} of production in period $t+1$ as

$$Y^e_{t+1}=aY_t,$$

where the coefficient a translates the subjective anticipation of the trend. Nevertheless, since our model is stationary and does not entail any technical progress, it is not unreasonable to privilege the case $a=1$.

If the production sector was exhibiting decreasing returns to scale (which may seem an awkward assumption with an aggregate production function using capital and labour as inputs), our procedure would differ from profit maximization. But it is clear from the partial equilibrium diagram that our procedure can converge while the assumption of profit maximization may lead to explosive behaviour of the Cobweb type.

Let us now turn to the study of the dynamics of the adjustment described above. The dynamic behaviour of the economy is driven by eqs. (9) and (10) which describe the evolution of capital from period to period. The whole sequence of temporary equilibria is exactly described by eqs. (2), (3), (4), (5), (9), (10), (11), and (12). We can now state

Proposition 1. The sequence of temporary equilibria (K_t, L_t, Y_t, S_t, C^1_t, C^2_t, w^*_t, ρ^*_t) *converges monotonically to a unique steady state as t tends to infinity. This steady state is the unique competitive equilibrium of the economy.*

The interest rate of the steady state is given by the equation

$$1+\rho^*_\infty = 1/\alpha(\rho^*_\infty) = \gamma K^{\gamma-1}\bar{L}^{1-\gamma}. \tag{13}$$

Notice that this equation implies the equality $\rho^*_\infty = r_\infty$; at the steady state, the interest rate is exactly equal to the marginal productivity of capital K_∞.

The other quantities and prices of the steady state as functions of ρ^*_∞ are given by the formulae

$$S_\infty = K_\infty = (\gamma/(1+\rho^*_\infty))^{1/(1-\gamma)}\bar{L}, \tag{14}$$

$$Y_\infty = (\gamma/(1+\rho^*_\infty))^{\gamma/(1-\gamma)}\bar{L}, \tag{15}$$

$$w^*_\infty = (1-\gamma)(\gamma/(1+\rho^*_\infty))^{\gamma/(1-\gamma)}, \tag{16}$$

$$C^1_\infty = (1-\gamma)(\gamma/(1+\rho^*_\infty))^{\gamma/(1-\gamma)}\bar{L}, \tag{17}$$

$$C^2_\infty = \rho^*_\infty(\gamma/(1+\rho^*_\infty))^{\gamma/(1-\gamma)}\bar{L}. \tag{18}$$

Two comments are in order. First the adjustment process whose convergence is proved in Proposition 1 can be viewed as a dynamic interpretation of the well-known distinction between short- and long-run total costs. This distinction is usually founded on the assumption that the level of fixed factors cannot be modified in the unit of time taken as reference for defining the flows of factors and products. Short-run costs of a given output level are derived from cost minimization of that output level at given prices when minimization operates only on the restricted set of those factors whose levels can be modified in the time unit (variable factors). By contrast long-run costs of the same output level obtain through cost minimization operating on all, included fixed, factors, and at the *same* prices. However extending from variable to fixed factors, the possibility of optimal adjustment of factor bundles can entail, as we have seen, fluctuations in their equilibrium prices, a property which may cast some doubt on the validity of the procedure of evaluating short- and long-run costs at the same factor prices. However in the steady state where factor prices remain constant through time, the concepts of short- and long-run costs are again meaningful.

Second, the convergence of our adjustment process to the steady state can be interpreted in terms of allocation efficiency. Since the set of values (K_∞, $\bar{L}$, C^1_∞, C^2_∞, Y_∞), combined with the prices (w^*_∞, ρ^*_∞), is the unique competitive equilibrium of our economy, the steady state constitutes a Pareto-optimal allocation. Our result can thus be viewed as describing an efficient method for decentralizing the adjustment of productive capacity when such adjustments become operational only with a one-period time lag on production decisions. At each period, the Planning Bureau instructs firms to adapt capacity with myopic anticipations about prices and output, so as to minimize the costs of producing the current output at those prices which clear instantaneously the labour and capital markets. Now we turn to the analysis of the same process if persistent wage rigidities are observed on the labour market.

3. The behaviour of the system when wages are persistently rigid

It is instructive to examine now, in the same model, the distortions introduced by wage rigidities in the labour market. Then the process moves on this market according to quantity, rather than wage, adjustments, and these reverberate in the investment decisions of the productive sector.

First we retain again Assumptions (A.1) and (A.2) so that, in particular, the supply of labour is still inelastic and equal to $\bar{L}$. But we assume now that the wage rate remains constant through time, and equal to $\bar{w}$. Denote by $L_t^{\mathrm{d}}(\bar{w})$ the demand for labour by the productive sector at the wage rate $\bar{w}$; this demand again obtains through short-run profit maximization, given the amount K_{t-1} of capital inherited from period $t-1$, i.e.,

$$L_t^{\mathrm{d}}(\bar{w}) = \left(\frac{1-\gamma}{\bar{w}}\right)^{1/\gamma} \cdot K_{t-1}.$$

Now two cases may arise. In the first one, $L_t^{\mathrm{d}}(\bar{w}) \geqq \bar{L}$, and the 'long side' of the market is the productive sector, which has to be rationed and can only get a quantity $\bar{L}$ of labour. In the second case, $L_t^{\mathrm{d}}(\bar{w}) < \bar{L}$; then the labour sector has to be rationed. *We shall restrict our attention to this second case only.*

We must then have the inequality

$$\left(\frac{1-\gamma}{\bar{w}}\right)^{1/\gamma} K_{t-1} < \bar{L}.$$

This inequality can be rewritten as

$$\left(\frac{w_\infty^*}{\bar{w}}\right)^{1/\gamma} \frac{K_{t-1}}{K_\infty} < 1,$$

where w_∞^* and K_∞ are the long-run equilibrium values of the process when no wage rigidity is observed [see (14) and (16)]. The equilibrium value for the quantity of labour exchanged on the labour market is then

$$L_t = \left(\frac{1-\gamma}{\bar{w}}\right)^{1/\gamma} K_{t-1}. \tag{19}$$

As for the capital we assume that the interest rate still adjusts instantaneously so as to equilibrate supply and demand on the market. Furthermore, we assume as before that, whatever the situation on the labour market, the demand for capital emanating from the productive sector is derived as described in the preceding section; firms minimize the costs of producing the observed output Y_t at current prices. Accordingly, the demand for capital of the productive sector is

$$K_t=\left(\left(\frac{\gamma}{(1-\gamma)}\right)^{1-\gamma}\bar{w}^{1-\gamma}\Big/(1+\rho_t)^{1-\gamma}\right)Y_t$$

[see (7) above with $w_t=\bar{w}$].

We can deduce from (19) the production in period t

$$\begin{aligned}&Y_t=K_{t-1}^{\gamma}[((1-\gamma)/\bar{w})^{1/\gamma}K_{t-1}]^{1-\gamma},\\&Y_t=((1-\gamma)/\bar{w})^{(1-\gamma)/\gamma}K_{t-1}.\end{aligned}\tag{20}$$

The demand of capital is therefore

$$K_t=\gamma^{1-\gamma}((1-\gamma)/\bar{w})^{(1-\gamma)^2/\gamma}K_{t-1}/(1+\rho_t)^{1-\gamma}.\tag{21}$$

To derive the savings of the capital owners, let us compute their revenue

$$R_t=Y_t-\bar{w}L_t,$$

$$R_t=\gamma((1-\gamma)/\bar{w})^{(1-\gamma)/\gamma}K_{t-1}.$$

Applying (A.2), the supply of capital is now equal to $\alpha(\rho_t)R_t$, i.e.,

$$S_t=\alpha(\rho_t)\gamma((1-\gamma)/\bar{w})^{(1-\gamma)/\gamma}K_{t-1}.\tag{22}$$

The equilibrium interest rate is determined by the clearing condition $S_t=R_t$, which gives

$$\alpha(\rho_t)(1+\rho_t)^{1-\gamma}=(1/\gamma^{\gamma})(\bar{w}/(1-\gamma))^{1-\gamma},\tag{23}$$

an equation independent of K_{t-1}. Let $\bar{\rho}$ denote the value of ρ_t which solves (23): with labour rationing, this equilibrium value of the interest rate remains constant through time and does not depend on K_{t-1}.

The capital saved for the next period is then

$$S_t=K_t=\alpha(\bar{\rho})\gamma((1-\gamma)/\bar{w})^{(1-\gamma)/\gamma}K_{t-1},$$

which can be written [see (16)] as

$$S_t = K_t = \alpha(\bar{\rho})(w^*_\infty/\bar{w})^{(1-\gamma)/\gamma}(1+\rho^*_\infty)K_{t-1}. \tag{24}$$

Finally let us prove that, even if equilibrium of the labour market is realized through quantity rationing, then equilibrium is spontaneously realized on the commodity market. The demand for the product coming from the labour sector is

$$C^1_t = \bar{w}L_t.$$

The revenue of capital owners obtains as

$$R_t = Y_t - \bar{w}L_t,$$

which is divided between their consumption C^2_t and savings S_t. We still have the identity

$$C^1_t + C^2_t + S_t = Y_t,$$

and there is no rationing on the commodity market.

We are now in position to study the sequence of temporary equilibria defined by eqs. (19), (20), (23), and (24). The dynamics are given by eq. (24) describing the motion of capital through time. The most symmetric form of this equation is

$$K_t = \frac{\alpha(\bar{\rho})}{\alpha(\rho^*_\infty)}\left(\frac{w^*_\infty}{\bar{w}}\right)^{(1-\gamma)/\gamma} K_{t-1}, \tag{25}$$

where $\bar{\rho}$ is defined by eq. (23). Clearly the coefficient of K_{t-1} does not depend on time.

Our major finding is that, if the wage rate $\bar{w}$ remains fixed at a value which exceeds w^*_∞ and if the initial stock of capital K_0 is such that there is unemployment in the first period, then the whole economy is driven down for ever on a destructive path.

Proposition 2. If $\bar{w} > w^*_\infty$ and if K_0 is such that the supply side of the labour market is rationed in period 1, i.e., K_0 verifies

$$\frac{w^*_\infty}{\bar{w}}\frac{K_0}{K_\infty} < 1,$$

then the values of K_t, L_t and Y_t decrease simultaneously to zero, at a constant rate which is a decreasing function of $\bar{w}$.

Thus Proposition 2 tells us a story very different from our Proposition 1. If the wage rate remains sticky at a level which exceeds the steady state wage rate w^*_∞, then the same adjustment process, which works so efficiently when no rigidity is observed, leads the economy to decline for ever at a constant rate. Enforcing the wage rate to exceed w^*_∞ reduces the demand for labour which in turn reduces production and savings. An 'under-investment gap' is created: the capacity level required to cancel the initial unemployment cannot be financed and unemployment grows worse. Of course this result should not be taken too seriously, to the extent that the assumption of complete wage rigidity is extreme and unrealistic. Most probably, the decrease in production and employment will exert a pressure on the wage rate to adjust to the ongoing conditions of the economy. A more elaborate version of the model should embody an equation reflecting this adjustment with the wage rate related to the rate of unemployment.

The analysis in the present section is certainly reminiscent of the contributions of Ito (1979) and Picard (1979) relative to Disequilibrium Growth Theory. These contributions aim at extending disequilibrium macroeconomics to the problem of capital accumulation. They show in particular that in a Solovian neoclassical growth model, long-run downward inflexibility of the wage rate may well result in a permanent decline of capital per capita [see, in particular, Ito (1979, p. 22)]. However their analysis deviates from ours in two respects. First, no time-lag is introduced between the effects of capacity adjustments and current production decisions, which is the essential feature of our analysis. Second, they perform their analysis under the assumption of constant propensity to save [$\alpha'(\rho)=0$], whereas we introduce explicitly the capital market, and let both the saving rate from profits and the demand for capital depend on the interest rate.[4]

4. Summary and conclusion

In this paper we have considered a simple model of a competitive economy, focusing our analysis on the problem of capacity adjustments resulting from a time-lag between production decisions and the availability of investments. Our main proposition has shown that a particular form of myopic behaviour of the productive sector about output and price anticipations combined with instantaneous price adjustments leads the economy to the efficient steady state where capacity is optimally adapted to the repeated

[4]The two analyses also deviate from each other on other points. Here no population growth is envisaged and the trajectories of capacity adjustments are only examined for the case where unemployment is experienced from the start. We have however have however verified that if our time-lag assumption is coupled with the assumption constant propensity to save, $\bar{w}>w^*_\infty$ *always* implies a permanent decline of capital, even if the starting capital endowments K_0 lead initially to an excess *demand* for labour at $\bar{w}$.

level of economic activity. By contrast, persistent wage inflexibility at a too high level generates a permanent decline of capacity. The assumption of naïve expectations w.r.t. the observed output level and prices is certainly a severe restriction, and may not be verified in many situations. However it captures the basic observation that firms move towards an increase in capacity if available capacity is too low relative to the realized output level, and a decrease in the opposite case. In any case this assumption is fruitful in our context since it provides at least an efficient method for decentralizing the adjustment of productive capacity. As it was suggested above, a possible extension would consist in introducing explicitly a representation of the anticipations of entrepreneurs about the evolution of the economy. These anticipations could be taken either as constant or, more interestingly, adaptive.

Our model is also particular in other respects. A Cobb–Douglas production function has been postulated; the whole saving activity is assumed to be concentrated in the hands of rentiers; and there is only a single type of fixed, and variable factor. We have tried, without success, to extend our analysis to a general constant returns to scale production function. The major difficulty encountered was that an explicit formula for the demand for capital [i.e., the analog of (7)] can no longer be derived if such a general form is adopted. Then *both* the supply and demand for capital are implicit, and it becomes impossible to prove the existence of a unique steady state of the process, a property which is crucially used in the proof of its convergence. However, as stated in footnote 3, if the assumption of constant saving fraction is introduced in place of (A.2), then our propositions extend to the whole class of constant returns to scale production functions satisfying the so-called 'Inada conditions'.

To dispense with the assumption that savings are performed from profit gains only seems also very hard. It would imply that income of the labourers does not reduce to wages only, and the budget constraint of the labour sector should be rewritten so as to take into account the future capital gains; but this creates the difficult issue of wealth accumulation. Perhaps an 'overlapping generations' model would be more appropriate in this context.

Finally, our analysis is limited by the fact that there is only a single consumption good which also serves for investment, and a single variable factor. A realistic representation would require an extension of the analysis to an arbitrary number of consumption goods and productive factors. Then a more complicated structure of time lags should be introduced, where the rough distinction between fixed and variable factors is replaced by varying degrees of 'rigidities' according to the factor considered: a 'heavy' investment, like building a new factory, may reveal its effects in two or three periods later, though deciding to buy a lot of new machines can become operational a single period later only. However the gains of realism obtained from such refinements would probably be heavily paid in terms of analytical tractability.

Appendix

Proof of Proposition 1. The process that we have to study is governed by the difference equation

$$K_t=\alpha(\rho_t^*(K_{t-1}))\gamma K_{t-1}^{\gamma}\bar{L}^{1-\gamma}, \tag{A.1}$$

where $\rho_t^*(K_{t-1})$ is the unique solution of the equation

$$\alpha(\rho)(1+\rho)^{1-\gamma}=\frac{1}{\gamma^{\gamma}}\frac{K_{t-1}^{\gamma(1-\gamma)}}{\bar{L}^{\gamma(1-\gamma)}}. \tag{A.2}$$

If we consider the function defined on $\mathbb{R}_+$ by

$$f(K)=\gamma\alpha(\rho^*(K))K^{\gamma-1}\bar{L}^{1-\gamma}, \tag{A.3}$$

the process can be rewritten as

$$K_t=K_{t-1}f(K_{t-1}).$$

To prove the convergence of the process, we will use the properties of the function f described in the following lemma:

Lemma. The function f defined by (A.3) is decreasing on $]0,+\infty[$ *with* $\lim_{K\to 0} f(K)=+\infty$ *and* $\lim_{K\to+\infty} f(K)=0$. *Furthermore the function* $Kf(K)$ *is increasing on* $]0,+\infty[$.

Proof of the Lemma. Since $0<\gamma<1$ and $0<\alpha(\rho)<1$, it is obvious that $\lim_{K\to+\infty} f(K)=0$.

To study the limit when $K\to 0$, let us rewrite eq. (A.2) which defines $\rho^*(K)$ as

$$K^{\gamma-1}\alpha(\rho^*(K))=\frac{1}{\gamma^{\gamma}}\frac{K^{-(1-\gamma)^2}}{(1+\rho^*(K))^{1-\gamma}\bar{L}^{\gamma(1-\gamma)}}.$$

This implies that $K^{\gamma-1}\alpha(\rho^*(K))\to 0$ when $K\to+\infty$, and then $\lim_{K\to+\infty} f(K)=0$.

To prove that f is decreasing, we compute $f'(K)$, i.e.,

$$f'(K)=\gamma K^{\gamma-2}\bar{L}^{1-\gamma}[\alpha'(\rho)\rho^{*\prime}(K)K+(\gamma-1)\alpha(\rho^*)].$$

Differentiating eq. A.2, we obtain

$$\alpha'(\rho^*)\rho^{*\prime}(K)(1+\rho^*)^{1-\gamma}+(1-\gamma)\alpha(\rho^*)\rho^{*\prime}(K)(1+\rho^*)^{-\gamma}=\frac{\gamma(1-\gamma)K^{\gamma(1-\gamma)-1}}{\gamma^{\gamma}\bar{L}^{\gamma(1-\gamma)}},$$

which implies $\rho^{*\prime}(K)>0$ [since by Assumption (*A.2*) $\alpha'(\rho^*)>0$].

Furthermore multiplying both sides of the equality by $K/\alpha(\rho^*)(1+\rho^*)^{1-\gamma}$, this equation rewrites [from (A.2)] as

$$\frac{\alpha'(\rho^*)\rho^{*\prime}(K)K}{\alpha(\rho^*)}+\frac{(1-\gamma)\rho^{*\prime}(K)K}{1+\rho^*(K)}=\frac{\gamma(1-\gamma)K^{\gamma(1-\gamma)}}{\alpha(\rho^*)\gamma^{\gamma}\bar{L}^{(1-\gamma)}(1+\rho^*)^{1-\gamma}}=\gamma(1-\gamma).$$

Since $\rho^{*\prime}(K)>0$, it follows that

$$\frac{\alpha'(\rho^*)\rho^{*\prime}(K)K}{\alpha(\rho^*)}<\gamma(1-\gamma).$$

Therefore

$$f'(K)<\gamma K^{\gamma-2}\bar{L}^{1-\gamma}[\gamma(1-\gamma)\alpha(\rho^*)+(\gamma-1)\alpha(\rho^*)]$$

$$\Leftrightarrow f'(K)<-\gamma(1-\gamma)K^{\gamma-2}\bar{L}^{1-\gamma}\alpha(\rho^*)<0.$$

Finally

$$Kf'(K)+f(K)=\gamma K^{\gamma-1}\bar{L}^{1-\gamma}[\alpha'(\rho^*)\rho^{*\prime}(K)K+(\gamma-1)\alpha(\rho^*)+\alpha(\rho^*)]$$

$$=\gamma K^{\gamma-1}\bar{L}^{1-\gamma}[\alpha'(\rho^*)\rho^{*\prime}(K)K+\gamma\alpha(\rho^*)]>0,$$

and the function $K\to Kf(K)$ is increasing. Q.E.D.

We can now study the process

$$K_t=K_{t-1}f(K_{t-1}).$$

First, there is a unique fixed point to this difference equation since the equation

$$K=Kf(K)$$

is equivalent to $f(K)=1$.

Since f is decreasing from $+\infty$ to 0, there is a unique value K_∞ which satisfies this equation.

To prove the convergence of the process to K_∞, let us prove that

$$K_{t-1} < K_\infty \Rightarrow K_{t-1} < K_t < K_\infty.$$

Since f is decreasing $K_{t-1} < K_\infty \Rightarrow f(K_{t-1}) > f(K_\infty) = 1$, which implies in turn $K_t = K_{t-1} f(K_{t-1}) > K_{t-1}$.

Since $K \to Kf(K)$ is increasing,

$$K_{t-1} < K_\infty \Rightarrow K_{t-1} f(K_{t-1}) < K_\infty f(K_\infty) \Leftrightarrow K_t < K_\infty.$$

Thus if $K_0 < K_\infty$, the sequence (K_t) is increasing and bounded above by K_∞. Accordingly, it must converge to K_∞ which is the unique solution of $K = Kf(K)$. Since a perfectly symmetric argument applies, mutatis mutandis, if $K_0 > K_\infty$, we get the desired convergence result.

Finally, to obtain the values at the steady state, let us remark that the equation

$$f(K_\infty) = 1$$

can be written as

$$\alpha(\rho^*(K_\infty)) \underset{\text{def}}{=} \alpha(\rho^*_\infty) = \frac{1}{\gamma}\left(\frac{K_\infty}{\bar{L}}\right)^{1-\gamma}. \quad \text{As}$$

$$\alpha(\rho^*_\infty)(1+\rho^*_\infty)^{1-\gamma} = \frac{1}{\gamma^\gamma}\left(\frac{K_\infty}{\bar{L}}\right)^{\gamma(1-\gamma)}, \quad \text{we obtain}$$

$$(1+\rho^*_\infty)^{1-\gamma} = \frac{1}{\gamma^{\gamma-1}}\left(\frac{K_\infty}{\bar{L}}\right)^{-(1-\gamma)^2} \quad \text{or} \quad 1+\rho^*_\infty = \gamma\left(\frac{\bar{L}}{K_\infty}\right)^{1-\gamma}$$

Therefore $\alpha(\rho^*_\infty)(1+\rho^*_\infty) = 1$, which is eq. (13), and $K_\infty = (\gamma/(1+\rho^*_\infty))^{1/(1-\gamma)}\bar{L}$, which is eq. (14). Then eqs. (15), (16), (17), (18) are obtained in an obvious way from eqs. (4), (5), (11), (12), and (14). Q.E.D.

Proof of Proposition 2. Assume that an excess supply of labour is observed at period 1. Then the process is from the start governed by eq. (25) with $K_{t-1} = K_0$. Consequently, if we prove that the coefficient of K_{t-1} in this equation is strictly smaller than one if $\bar{w} > w^*_\infty$, we shall obtain $K_1 < K_0$, so that the inequality $(w^*_\infty/\bar{w})^{1/\gamma} \cdot (K_1/K_\infty) < 1$ should also hold: an excess supply of labour is again observed at period 2; repeating the argument, the amount of capital would be monotonically decreasing and the process remains for ever governed by eq. (25). Thus the proof of the first part of our proposition is complete if we prove that

$$\bar{w} > w_\infty^* \Rightarrow \frac{\alpha(\bar{\rho})}{\alpha(\rho_\infty^*)} \cdot \left(\frac{w_\infty^*}{\bar{w}}\right)^{(1-\gamma)/\gamma} < 1,$$

with $\bar{\rho}$ the solution of (23), i.e., such that $\alpha(\bar{\rho})(1+\bar{\rho})^{1-\gamma} = (1/\gamma^\gamma)(\bar{w}/(1-\gamma))^{1-\gamma}$ (notice that ρ_∞^* is the solution of this equation if $\bar{w} = w_\infty^*$).

To that effect denote by $\rho(w)$ the solution of the equation

$$\alpha(\rho)(1+\rho)^{1-\gamma} = \frac{w^{1-\gamma}}{\gamma^\gamma(1-\gamma)^\gamma}, \tag{A.4}$$

and let $h(w)$ be defined by

$$h(w) \underset{\text{def}}{=} \frac{\alpha(\rho(w))}{w^{(1-\gamma)/\gamma}}.$$

With this notation, the difference eq. (25) rewrites as

$$K_t = \frac{h(w)}{h(w^*)} K_{t-1}.$$

If we prove that $h'(w) < 0$, then this will prove first that the coefficient of K_{t-1} is strictly smaller than one, and also that it decreases with $\bar{w}$. Accordingly the proof of Proposition 2 is complete if we show $h'(w) < 0$. An immediate computation shows that $h'(w)$ has the same sign as u with u defined by

$$u(w) \underset{\text{def}}{=} \alpha(\rho) \cdot \rho' - \frac{1-\gamma}{\gamma} \frac{\alpha(\rho)}{w}. \tag{A.5}$$

Substituting in (A.5) the value of $\alpha(\rho)$ obtained from (A.4), (A.5) rewrites as

$$u(w) = \alpha'(\rho) \cdot \rho' - \frac{(1-\gamma)^\gamma}{\gamma \cdot \gamma^\gamma \cdot w^\gamma} \cdot \frac{1}{(1+\rho)^{1-\gamma}} \tag{A.6}$$

or

$$(1+\rho)^{1-\gamma} \cdot u(w) = \alpha'(\rho) \cdot \rho' \cdot (1+\rho)^{1-\gamma} - \frac{1}{\gamma}\left[\frac{(1-\gamma)^\gamma}{\gamma^\gamma \cdot w^\gamma}\right].$$

Differentiating totally (A.6) we obtain

$$\alpha'(\rho) \cdot \rho' \cdot (1+\rho)^{1-\gamma} + (1-\gamma) \cdot \alpha(\rho)(1+\rho)^{-\gamma} \cdot \rho' = \frac{(1-\gamma)^\gamma}{\gamma^\gamma \cdot w^\gamma}. \tag{A.7}$$

From (A.7) we deduce

$$(1+\rho)^{1-\gamma}\cdot u(w)=\frac{(1-\gamma)^{\gamma}}{\gamma^{\gamma}\cdot w^{\gamma}}\left(1-\frac{1}{\gamma}\right)-(1-\gamma)\cdot\alpha(\rho)\cdot(1+\rho)^{-\gamma}\cdot\rho'<0,$$

where the last inequality follows from the fact that $1-1/\gamma<0$ and $\rho'>0$ [solve ρ' in eq. (A.7) and keep in mind that Assumption (A.2), $\alpha'>0$]. Thus $u(w)<0$ so that $h'(w)<0$, and the proof is complete. Q.E.D.

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